Intersectoral demand linkages: Good shocks, bad outcomes?*

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When and how are sector-specific price shocks magnified or dampened in general equilibrium with multiple industries and distortions? We develop a general framework with homothetic sectoral preferences and derive a welfare multiplier, which is a sufficient statistic for the share of the direct effect of the shock that materializes in the aggregate. We show that the combination of Cobb-Douglas or CES preferences with monopolistic competition always yields welfare gains from a positive shock. However, a positive sectoral shock may lead to aggregate losses under departures from either CES preferences or monopolistic competition. While our approach is similar to Baqaee and Fahri (2019, 2024), shocks propagate via consumer preferences in our case and production networks in their case, and market structure plays an explicit role in the transmission of shocks in our model.

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1 Introduction

Assessing the welfare effects of economic shocks in the presence of distortions is of great importance in various applied fields of economics. While many models zoom onto selected distortions to provide insights into specific channels, their main focus is generally to assess how the gap between the equilibrium and the optimum changes following a shock.[1] Most of those models also consider a single industry only or provide a partial equilibrium analysis. Our paper, on the contrary, seeks to understand whether and how sector-specific price shocks—that is, shocks that only directly affect prices in a single sector but also indirectly affect prices in other sectors due to how consumers reallocate their expenditure between sectors—are magnified or dampened in a general equilibrium context with multiple industries and distortions. It is thus in line with recent contributions by Baqaee and Fahri (2019, 2024) who, following the pioneering work by Hulten (1978), investigate the transmission of localized microeconomic shocks to the macroeconomy in the presence of various distortions. Do the welfare effects of positive sectoral shocks get amplified or dampenend in general equilibrium? Could they even get reversed? What mechanisms are at play? And are there sufficient statistics to measure them?

Answering these questions is relatively straightforward in efficient economies that satisfy the fundamental welfare theorem. In this case, because the equilibrium outcome maximizes welfare, the interactions between sectors cancel out. Put differently, by the envelope theorem, only the direct effect of the shock in question matters for welfare. This line of reasoning usually no longer applies to distorted economies with, e.g., imperfect competition. The general theory of the second-best (Lipsey and Lancaster, 1956) tells us that, in the presence of multiple distortions, a shock to one sector need not be welfare improving even if it reduces distortions in the sector that experiences the shock. With imperfect competition and multiple distortions we thus need to worry about a richer array of general-equilibrium effects channeled through sectoral differences in monopoly power, firms' market conduct, product differentiation, and the presence of various entry- and exit barriers.

The aim of this paper is threefold. First, we develop a general framework with homothetic sectoral preferences that nests many of the models used in the applied literature. Within that framework, we consider the welfare effects of a positive shock that directly reduces the price level in a single sector. This decrease in the sectoral price level could be driven by a variety of microeconomic mechanisms such as increased productivity, decreased trade costs, or decreased

[1]See, e.g., Dixit and Stiglitz (1977) for the private versus social value of variety; Dixit and Norman (1980, Ch.9) for non-optimal product selection; Mankiw and Whinston (1986) for the business stealing effect and differences between free- and restricted-entry equilibria; Dhingra and Morrow (2019) for potential misallocations between heterogeneous firms; Francois and van Ypersele (2002) and Janeba (2007) for trade in 'cultural goods' subject to network externalities; and Epifani and Gancia (2011) and Behrens *et al.* (2020) for misallocations between heterogeneous industries and firms. Vives (1999) provides a detailed overview of different imperfectly competitive models and their positive and normative results.

fixed costs (as in Arkolakis et al., 2012, 2019; and Baqaee and Farhi, 2024).[2] To be precise, for *a given sectoral allocation*, the shock we consider leads to welfare gains. We refer to this as the *direct effect* of the shock. However, in a multi-sector economy, the shock leads to reallocations of consumer budgets across sectors, i.e., changes in sectoral market sizes. The latter, in turn, impact welfare through changes in sectoral price levels via changes in firms' individual prices and/or the range of available varieties. We refer to the latter as the *indirect effect* (or general equilibrium effect) of the sectoral shock that propagates through the economy.

We then derive a welfare multiplier, $\mathcal{M}$, which is a *sufficient statistic* that measures the share of the direct effect of the positive shock that materializes in general equilibrium. Using $\mathcal{M}$, we establish conditions under which a sectoral shock is magnified ($\mathcal{M} > 1$), equal to the direct effect ($\mathcal{M} = 1$), dampened ($0 \leq \mathcal{M} < 1$), or even reversed ($\mathcal{M} < 0$) in the aggregate. The latter case can only occur when goods are gross substitutes.

We provide clear-cut intuition for our results with two sectors, in which case the magnitude and the sign of the indirect effect depend on the complementarity or substitutability between the sectors in consumers' preferences.[3] We show that if the shock affects the sector with the more elastic price index with respect to consumers' budget, the welfare gains are magnified when the two goods are gross substitutes; whereas they are dampened if the goods are gross complements. The intuition is that when goods are gross substitutes, the reduction in prices in the sector hit by the shock leads to an increase in its market size as consumers reallocate budget to that sector. The latter amplifies the direct effect via either lower markups, entry of additional firms, or a combination of both, which further reduces the price level, thereby magnifying the effect of the initial shock. Conversely, the market size of the sector not directly exposed to the shock shrinks as consumers reallocate budget from that sector, which leads to higher markups, exit of firms, or a combination of both, thus dampening the direct effect of the initial shock. When the shock originates in the sector with the more elastic price index, the positive indirect effect in that sector always outweighs the negative indirect effect in the other sector.

Our second contribution is to show that a specific combination of assumptions widely used in the literature—namely Cobb-Douglas or CES preferences and a monopolistically competitive market structure—give rise to very specific results. To be precise, we show that $\mathcal{M} = 1$ holds with Cobb-Douglas preferences across sectors: since a sectoral shock has no effect on consumers' budget allocation, there are no intersectoral dependencies and thus no indirect effects.[4] We further show that if both sectors are of the CES-monopolistic competition type, and for an arbitrary

[2]While our general framework is agnostic about the origin of the shock, we consider different micro foundations in our applications.

[3]Substitutes and complements are generally only clearly defined at the level of a good when there are two goods only. With more than two goods, we can no longer talk about goods being complements or substitutes, only about pairs being substitutable or complementary. This leads to a combinatorially exploding number of different cases to consider. We return to this point when deriving more general multi-sector results.

[4]Baqaee and Farhi (2019) make the same point regarding Cobb-Douglas production functions in a model with production networks.

upper-tier utility function, a price-reducing shock (for example, trade liberalization or productivity improvements) in the first sector always has positive aggregate welfare effects, provided the cross-sectoral elasticity of substitution is lower than the elasticities of substitution within sectors. The reason is that prices are invariant to changes in sectoral market sizes in the CES model, so that changes in price indices are entirely driven by changes in the sectoral masses of firms. Even when the indirect effect is negative—which as explained above is the case when the price index in the first sector hit by the shock is less elastic than the price index in the other sector—the welfare losses due to firm exit in the second sector are never strong enough to overcome the direct welfare-improving effect in the first sector. In other words, a positive sector-specific shock always translates into aggregate welfare gains under 'reasonable' assumptions on the elasticities of substitution of different tiers of preferences. We illustrate this finding using two examples, one where the shock corresponds to a trade liberalization à la Krugman (1980)—a decline in iceberg trade costs in one of the sectors—in a two-country open economy, and one where the shock corresponds to a left-shift of the cost distribution in a setting with heterogeneous firms à la Melitz (2003). In both cases, the shock must improve aggregate welfare.

Third, we show that there exists a large class of homothetic non-CES preferences and different market structures under which a positive sector-specific shock may lead to aggregate welfare losses—even when the cross-sectoral elasticity of substitution is lower than the elasticities of substitution within sectors—when goods are gross substitutes. Starting with monopolistic competition and non-CES preferences, we show that with homothetic variable elasticity of substitution (VES) preferences, prices vary with the sectoral market sizes. Therefore, a positive shock to one sector forces firms out of the other sector which relaxes competition and, ultimately, increases prices in that sector. A rise in prices is an additional negative effect that reinforces the loss of product variety in that sector. If the elasticity of the sectoral price level with respect to market size is sufficiently large, this negative effect is strong enough so that overall welfare decreases.

Turning to market structures that differ from monopolistic competition, but keeping CES preferences in both sectors, we show that a combination of oligopoly and CES preferences gives rise to price indices that vary with sectoral market size because the elasticity of demand does not equal the elasticity of substitution. As a result, a positive shock to one sector produces negative effects via higher prices (markups) and less variety in the sector not exposed to the shock. These negative effects may dominate the positive price and variety effects in the sector exposed to the shock. This is, for example, likely to occur when the sector not exposed to the shock is sufficiently 'granular' (i.e., there are a small number of competing firms) such that firm exit produces substantial negative welfare effects.[5] Hence, both non-CES preferences and market structure matter substantially for assessing welfare changes in multisector models with

[5]This discussion shows that the mechanism is different from that in the immiserizing growth literature (Johnson, 1955; Bhagwati, 1958) and the 'Dutch desease' (Corden and Neary, 1982). Our effects are not driven by adverse changes in the terms of trade in international markets but go through consumer preferences only.

imperfect competition (see also Behrens *et al.*, 2020; d'Aspremont and Dos Santos Ferreira, 2016).

Our paper is mainly linked to three strands of literature. First, it is linked to the literature on welfare in imperfectly competitive markets with multiple industries (e.g., Epifani and Gancia, 2011; d'Aspremont and Dos Santos Ferreira, 2016; Behrens *et al.*, 2020). It complements that literature by deriving sharper conditions on how intersectoral effects affect welfare changes due to sector-specific shocks. It also complements contributions that deal with the positive analysis of multisector monopolistic competition models (e.g., Matsuyama, 1995).[6] Second, it is linked to the international trade literature with multiple industries and product differentiation. While some of that literature deals with the positive aspects only—e.g., trade elasticities in Ossa (2015) or changes in productivity cutoffs in Segerstrom and Sugita (2015)—another part of that literature sets out to explicitly quantify the welfare effects (e.g., Hsieh *et al.*, 2016). Last, our model is related to the recent literature that seeks to understand the transmission of localized microeconomic shocks to the macroeconomy following Baqaee and Fahri (2019, 2024). We highlight the similarity between consumer preferences—where shocks are transmitted between sectors via budget shares—and production network analysis—where shocks are transmitted between sectors through production networks. Formally, the two approaches are very similar, safe for the transmission channel: consumers in our case, and producers in Baqaee and Fahri (2019, 2024). However, a major difference with Baqaee and Farhi (2019, 2024) is that one should be agnostic about market structure in their models. In their case, the substitution patterns across a fixed range of products are informed by the data, and the market structure that has generated those patterns is irrelevant and taken as given. In our approach, instead, substitution patterns are the endogenous outcome of differences in market structures. As we show, both differences in preferences and differences in market structures may generate the same behavior of substitution patterns, thus suggesting that market structure is equally important in analyzing how shocks propagate through the economy.

The remainder of the paper is organized as follows. Section 2 presents two simple motivating examples that illustrate how good shocks can have different welfare consequences depending on small variations in modeling choices. Section 3 presents our general two-sector model, establishes the equilibrium, and derives the welfare multiplier that is a sufficient statistic to subsume the welfare effects of sector-specific shocks. We focus on interior equilibrium, which is essential for our analysis because in case of corner equilibria, only one sector is active, and thus, there are no intersectoral demand linkages anymore. In that case, $\mathcal{M} = 1$ and all effects are direct. Section 4 presents multiple applications of our general results. Section 5 extends our analysis to an arbitrary number of sectors. Section 6 concludes. Proofs and additional material are relegated to a set of appendices.

[6] We may view our paper as a 'normative companion' to Matsuyama's (1995) positive analysis of monopolistic competition models. While Matsuyama (1995) does not directly investigate welfare, he analyzes two-sector models and discusses some efficiency considerations.

2 Introductory examples

The aim of this section is to introduce the key ideas underlying our results in the simplest possible setting to lay the foundations for the more complete analysis to follow. We consider a single country with a unit mass of identical consumers and two sectors, 1 and 2. The utility of the representative consumer is given by

$$U(X_1, X_2), \tag{1}$$

where X_i is the subutility from the consumption of sector i's products. We assume that X_i is homothetic, so that a well-defined price index P_i exists.[7] By construction, the budget constraint can be written as $P_1X_1 + P_2X_2 = y$, where y denotes income. We let $W = V(P_1, P_2, y)$ stand for the indirect utility (welfare) of the representative consumer.

Demand linkages. To keep things simple, assume that preferences (1) can be expressed as

$$U(X_1, X_2) = \left(X_1^{\frac{\gamma-1}{\gamma}} + X_2^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}, \tag{2}$$

where $\gamma > 0$ is the constant intersectoral elasticity of substitution. Maximizing (2) subject to the budget constraint, the budget shares are as follows:

$$a(P_1, P_2) = \frac{P_1X_1}{y} = \frac{P_1^{1-\gamma}}{P_1^{1-\gamma} + P_2^{1-\gamma}}, \qquad 1 - a(P_1, P_2) = \frac{P_2X_2}{y} = \frac{P_2^{1-\gamma}}{P_1^{1-\gamma} + P_2^{1-\gamma}}. \tag{3}$$

As shown by (3), the budget allocation across sectors depends solely on P_1 and P_2. Their elasticities with respect to the price indices are given by

$$\mathcal{E}_{P_1}(a) = (1-\gamma)(1-a), \quad \text{and} \quad \mathcal{E}_{P_2}(1-a) = (1-\gamma)a, \tag{4}$$

the sign of which depends on whether $\gamma > 1$ (meaning X_1 and X_2 are gross substitutes) or $\gamma < 1$ (that is, X_1 and X_2 are gross complements). We let $y \equiv 1$ by choice of numéraire, welfare is

$$W = V(P_1, P_2) = \left[\left(\frac{a}{P_1}\right)^{\frac{\gamma-1}{\gamma}} + \left(\frac{1-a}{P_2}\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}. \tag{5}$$

Assume that both sectors produce final consumption goods and that there are no technological linkages. Sector 1 is perfectly competitive and produces a homogeneous good with constant marginal cost c_1, so that $P_1 = c_1$. Sector 2 is monopolistically competitive and produces a composite good represented by a continuum $[0, N]$ of differentiated varieties. The subutility X_2 and

[7] With homothetic preferences, the indirect utility X_i is a special case of the Gorman polar form and can, therefore, be represented by an ideal price index P_i (see Gorman, 1961; Jehle and Reny, 2011).

the price index P_2 are given by the standard CES aggregators:

$$X_2 = \left(\int_0^N x_i^{\frac{\sigma_2 - 1}{\sigma_2}} di \right)^{\frac{\sigma_2}{\sigma_2 - 1}}, \quad P_2 = \left(\int_0^N p_i^{1-\sigma_2} di \right)^{\frac{1}{1-\sigma_2}},$$

where x_i and p_i are, respectively, the quantity consumed and the market price of variety $i \in [0, N]$, while $\sigma_2 > \max\{1, \gamma\}$ is the intrasectoral elasticity of substitution. In other words, varieties of good 2 are better substitutes than goods 1 and 2. Each variety i is produced by one and only one firm, also labelled i, with constant marginal cost c_2 identical across firms and a fixed production cost $f_2 > 0$.

Consider a positive productivity shock in sector 1 which reduces costs: $\mathrm{d}c_1 < 0$. Any such shock has a direct effect of lowering the price $P_1 = c_1$, and an indirect effect created by reallocating consumers' budget between sectors (except for the Cobb-Douglas case, $\gamma = 1$, where budget shares are constant). The indirect effect can never more than offset the positive direct effect so that consumer welfare increases.[8] However, it is not clear whether the positive shock in sector 1 gets amplified or dampened. Furthermore, in the absence of perfect competition it may even get reversed, i.e., an a priori welfare-improving shock to one sector may lead to aggregate welfare losses.

Using the standard CES demand $x_i = \frac{1-a}{P_2}\left(\frac{p_i}{P_2}\right)^{-\sigma_2}$, firm i's profit maximization problem is given by

$$\max_{p_i} \ \pi_i = (p_i - c_2)\frac{1-a}{P_2}\left(\frac{p_i}{P_2}\right)^{-\sigma_2} - f_2,$$

so that standard monopoly pricing rule implies $p_i = c_2\sigma_2/(\sigma_2 - 1)$ for all $i \in [0, N]$. The endogenous mass N of varieties is pinned down by the the zero-profit condition,

$$\pi_i = 0 \quad \Longrightarrow \quad N = \frac{1-a}{\sigma_2 f_2},$$

while the price index P_2 becomes

$$P_2 = \frac{\sigma_2 c_2}{\sigma_2 - 1} N^{-\frac{1}{\sigma_2 - 1}} = \frac{\sigma_2 c_2}{\sigma_2 - 1}\left(\frac{1-a}{\sigma_2 f_2}\right)^{-\frac{1}{\sigma_2 - 1}}. \tag{6}$$

What are the effects of a shock to c_1 on welfare? To see this, we set $y = 1$ in (3) and totally log-differentiate it with respect to c_1. Using (4),

$$\frac{d\ln(1-a)}{d\ln c_1} = -\frac{a}{1-a}\mathcal{E}_{P_1}(a) + \mathcal{E}_{P_2}(1-a)\frac{d\ln P_2}{d\ln c_1} = a(\gamma - 1)\left(1 - \frac{d\ln P_2}{d\ln c_1}\right).$$

[8] In the absence of demand linkages, the decrease in c_1 strictly reduces P_1 without affecting P_2. Since the initial consumption bundle remains in the new budget set, the new optimal solution must yield strictly higher welfare.

This, combined with log-differentiating both sides of (6) with respect to c_1, yields:

$$\frac{d\ln P_2}{d\ln c_1} = -\frac{1}{\sigma-1}\frac{d\ln(1-a)}{d\ln c_1} = a\frac{1-\gamma}{\sigma-1}\left(1-\frac{d\ln P_2}{d\ln c_1}\right).$$

Hence,

$$\frac{d\ln P_2}{d\ln c_1} = \frac{a\,(1-\gamma)}{\sigma_2-1+a\,(1-\gamma)}, \tag{7}$$

which is negative if the goods produced by the two sectors are gross substitutes ($\gamma > 1$), and positive for gross complements ($\gamma < 1$).

Intuitively, gross substitutability/complementarity leads to lower/higher welfare gains than those stemming from the direct effect only. Indeed, plugging (3) into (5), the welfare becomes $W = \left(P_1^{1-\gamma} + P_2^{1-\gamma}\right)^{\frac{1}{\gamma-1}}$, and thus

$$\frac{\mathrm{d}W}{\mathrm{d}c_1} = \frac{\partial W}{\partial c_1}\mathcal{M}, \quad \text{with} \quad \frac{\partial W}{\partial c_1} = a\frac{W}{c_1} \quad \text{and} \quad \mathcal{M} \equiv \frac{1}{1-\frac{1-\gamma}{\sigma_2-\gamma}\,(1-a)}. \tag{8}$$

Expression (8) shows that the welfare effects of the shock depend on the direct effect, $\frac{\partial W}{\partial c_1}$, magnified or dampened by a *welfare multiplier* $\mathcal{M}$, whose structure is somewhat similar to the familiar Keynesian multiplier. When the goods produced by the two sectors are gross complements ($\gamma > 1$), we have $\mathcal{M} > 1$ (recall that $\sigma_2 > \gamma$), meaning that the multiplier amplifies the direct welfare effect. On the contrary, under gross substitutes ($\gamma < 1$), we have $0 < \mathcal{M} < 1$, meaning that there is a negative indirect effect. The latter is, however, never stronger than the direct effect, i.e., there are aggregate welfare gains. However, the indirect effect mitigates those gains under $\gamma > 1$ and those gains vanish as γ approaches σ_2, that is, when the gap between the inter- and intra-sectoraltoral elasticities of substitution shrinks.

The foregoing example is admittedly simple and entirely driven by the demand linkages between sectors as captured by the intersectoral elasticity of substitution γ. In the absence of that linkage (i.e., $\gamma = 1$, which is the Cobb-Douglas case), $\mathcal{M} = 1$ and only the direct effect of the shock matters for welfare. Showing how to derive $\mathcal{M}$—which is expressed in terms of the elasticities of prices and budget shares only—in the more general case is one of the key objectives of our paper.

The welfare multiplier $\mathcal{M}$ can be viewed as a demand-side counterpart of the network multipliers generated by technological linkages. The growing literature on production networks has investigated the effects of input-output links, as in Baqaee and Farhi (2019, 2020), on aggregate welfare outcomes. To illustrate the similarities between our approach and the one based on technological externalities, consider the following example.

Technological linkages. Assume that both sectors are competitive. Let Q_i denote the aggregate output of sector i. Marginal cost in sector 1 is constant and given by c_1. Sector 1 generates a technological externality for sector 2: we assume that marginal cost in sector 2 is a function of sector 1's output, i.e., $c_2 = c(Q_1)$. If sector 1 is, e.g., manufacturing and sector 2 is agriculture, there could be $\cup$-shaped relationship for $c(\cdot)$: more manufacturing is initially good for agriculture, because it provides better tools and allows for lower costs; but it is bad later because of increasing environmental degradation. An alternative interpretation could be the existence of some agglomeration economies.

For an arbitrary upper-tier utility (except Cobb-Douglas), the output of sector 1 depends on prices for both goods, $Q_1 = Q_1(P_1, P_2)$. With perfect competition, we have $P_1 = c_1$ and $P_2 = c_2 = c(Q_1)$, so that

$$c_2 = c\big(Q_2(P_2, c_1)\big) \quad \Longrightarrow \quad P_2 = c\big(Q_1(P_2, c_1)\big) \tag{9}$$

From (9), we thus have $P_2 = f(c_1)$ for some f, and equilibrium welfare $W = V(c_1, f(c_1), y)$, where V stands for the indirect utility.

Any shock to c_1 has a direct effect on welfare via sector 1, and an *indirect effect on welfare via sector* 2 because of intersectoral technological linkages. Assume a positive productivity shock in sector 1, i.e., $\mathrm{d}c_1 < 0$. The welfare change is:

$$\frac{\mathrm{d}W}{\mathrm{d}c_1} = \underbrace{\frac{\partial V(c_1, f(c_1), y)}{\partial P_1}}_{\text{direct effect}} + \underbrace{\frac{\partial V(c_1, f(c_1), y)}{\partial P_2} f'(c_1) + \frac{\partial V(c_1, f(c_1), y)}{\partial y} \frac{\mathrm{d}y}{\mathrm{d}c_1}}_{\text{indirect effect}},$$

which depends on the direct effect of the shock and the effects induced by intersectoral linkages and changes in income. Profits being zero, while $y \equiv 1$ by the choice of numéraire, so that $\mathrm{d}y/\mathrm{d}c_1 = 0$. We then have (suppressing function arguments to alleviate notation):[9]

$$\begin{aligned} \frac{\mathrm{d}W}{\mathrm{d}c_1} &= \frac{\partial V}{\partial P_1}\left[1 + \frac{\partial V/\partial P_2}{\partial V/\partial P_1} f'(c_1)\right] = \frac{\partial V}{\partial P_1}\left[1 + \frac{X_2}{X_1} f'(c_1)\right] \\ &= \frac{\partial V}{\partial P_1}\left[1 + \frac{a^*}{1-a^*}\mathcal{E}_{c_1}(P_2)\right] = V_1 \mathcal{M}, \end{aligned} \tag{10}$$

where $\mathcal{E}_{c_1}(P_2) = \frac{f'(c_1)c_1}{f(c_1)}$ is the elasticity of sector 2's price with respect to sector 1's marginal cost.

Expression (10) shows that the welfare effects of the shock depend on the direct effect, V_1, magnified or dampened by a *welfare multiplier* $\mathcal{M}$. It follows from (10) that

$$\mathcal{M} \gtreqless 1 \quad \Longleftrightarrow \quad \mathcal{E}_{c_1}(P_2) \gtreqless 0 \qquad \text{and} \qquad \mathcal{M} > 0 \quad \Longleftrightarrow \quad \mathcal{E}_{c_1}(P_2) > -\frac{1-a}{a}. \tag{11}$$

[9] The second equality is obtained using Roy's identity, which implies that $\frac{\partial V/\partial P_2}{\partial V/\partial P_1} = \frac{(\partial V/\partial P_2)/(\partial V/\partial y)}{(\partial V/\partial P_1)/(\partial V/\partial y)} = \frac{X_1}{X_2}$. The third equality uses the budget share $a(P_1, P_2) = a(c_1, f(c_1)) = P_1 X_1$.

If $\mathcal{E}_{c_1}(P_2) > 0$, there are welfare gains following a positive productivity shock to sector 1. The intuition is that the positive shock, on top of directly reducing prices in sector 1 also leads to lower prices in sector 2. However, when $\mathcal{E}_{c_2}(P_1) < 0$, welfare gains of a positive shock to sector 1 are always dampened since prices in sector 2 increase. If this effect is strong enough, welfare could even fall following the positive shock.

In what follows, we will derive a welfare multiplier $\mathcal{M}$ akin to the one introduced above and show which underlying elasticities determine whether the welfare gains of a shock are amplified, dampened, or even reversed in some cases. Although we focus mostly on two sectors, to provide clear-cut results, we extend the analysis later to the multi-sector case.

3 Two industries: General results

We first derive general results for the case of a closed economy with two sectors. We illustrate the case of an open economy in Section 4.1.1, and discuss the case with more sectors—and the relation between our work and existing multisector models—in Section 5.

We assume that there is a unit mass of identical consumers who have *weakly separable* preferences across the differentiated goods (Blackorby *et al.*, 1983). Hence, the utility function can be represented as $U(X_1, X_2)$, where $U : \mathbb{R}^2_+ \to \mathbb{R}$ is an *upper-tier* utility and X_s for $s = 1, 2$ are *subutilities* defined over the set of varieties produced in each sector. We assume that U is continuous, strictly increasing, and strictly quasi-concave. As to the subutilities, we assume that they are homothetic. Hence, the indirect utility X_s is a special case of the Gorman polar form and admits an ideal price index P_s.

We are for now agnostic about the market structure and production technology in each sector, except that we assume that no profits are redistributed in the economy.[10] As in the motivating examples, labor is the numéraire and we normalize income to one. Let α denote any budget share for sector 1 good (which we refer to as good 1), and $a = a(P_1, P_2)$ the optimal budget share—from the consumer's utility maximization problem—for good 1. Under our assumptions, the consumer's optimization problem can be solved using two-stage budgeting (Jehle and Reny, 2011), the budget share a being the solution to the first stage. Since income is normalized to one, we have $a(P_1, P_2) \equiv \arg\max_\alpha U(\alpha/P_1, (1-\alpha)/P_2)$, where $\alpha/P_1 = X_1$ and $(1-\alpha)/P_2 = X_2$ are the representations of the indirect utilities in the two sectors.

As in Matsuyama and Ushchev (2017), the mapping $a(\cdot, \cdot)$ summarizes all the information we need to determine equilibrium and sign welfare changes, without knowing U. When goods are gross substitutes, the budget share mapping $a(P_1, P_2)$ and $1 - a(P_1, P_2)$ is a *primitive of the model* in the following sense:

[10] This is, e.g., the case with perfect competition, or imperfect competition and free entry and exit, or absentee shareholders. The important point for our analysis is that profits do not feed back into the consumers' budget.

Lemma 1 (Primitive of the model)**.** Any differentiable function $a(P_1, P_2)$ that: (i) decreases in P_1; (ii) increases in P_2; and (iii) maps all price vectors $(P_1, P_2) \in \mathbb{R}_+^2$ into $[0, 1]$ is a budget share of good 1 generated by some monotonic, continuous, and strictly quasi-concave utility function defined over $\mathbb{R}_+^2$ in which goods are gross substitutes.

Proof. See Appendix A. □

Note that Lemma 1 holds only for the case of gross substitutes. Hence, $a(P_1, P_2)$ may not be a primitive of the model when goods are gross complements. This latter case is, however, much more straightforward to analyze.

3.1 Equilibrium

With two-stage budgeting, we can determine *intrasectoral equilibria* for given budget shares α and $1 - \alpha$. These consist of sectoral price indices $P_1(\alpha; \theta)$ and $P_2(1 - \alpha)$ that are functions of sector 1's budget share α and a shifter θ. The latter affects the price index in sector 1 only. In this section, we do not specify the nature of this shifter (shock) to keep the analysis general. The only assumption we make is that its direct effect is to solely decrease the price index in sector 1. However, in the following sections we provide a number of examples for such shocks which could be sectoral trade liberalization, productivity improvements, or a reduction in fixed costs in markets with free entry. We finally assume that P_1 and P_2 are continuous in α and θ.

Computing the price indices explicitly requires to make assumptions about market structure and technology.[11] For now, we abstract from these and simply assume that price indices representing an intrasectoral equilibrium exist. We further impose some minimal assumptions on these price indices: **(A1)** $\partial P_1/\partial\theta < 0$; and **(A2)** $\partial P_1/\partial\alpha < 0$ and $\partial P_2/\partial(1-\alpha) < 0$. Assumption (**A1**) states that—conditional on the budget share α—an increase in θ makes the products of the first sector less expensive. It follows that the direct effect of increasing θ is welfare improving. Assumption (**A2**) states that a sector's price index falls when the budget share allocated to that sector increases. The intuition is that when consumers spend more on the goods produced by a particular sector, this eventually reduces the price level in that sector (e.g., because of entry and the associated pro-competitive effects, or because of efficiency gains in production).[12] Note that the price indices in Section 2 satisfy (**A1**) and (**A2**). We will provide additional examples of micro-found price indices with such behavior in Section 4.

We are now equipped to formally define an equilibrium:

[11] Depending on the context, these intrasectoral equilibria may be: (i) a monopolistically competitive equilibrium; (ii) a Cournot-Nash equilibrium with entry; (iii) a Bertrand-Nash equilibrium with entry; or (iv) a more complex equilibrium concept, e.g., with varying toughness of competition à la d'Aspremont and Dos Santos Ferreira (2009).

[12] Let us emphasize that (**A2**) is not an ad hoc assumption. It holds for free-entry market structures such as monopolistic competition and oligopoly with both increasing and constant elasticity of demand, as well as with decreasing elasticity of demand when anti-competitive effects are mild. These assumptions are in line with the empirical evidence that documents the existence of pro-competitive effects (e.g., Bellone *et al.*, 2016).

Definition 1. *An equilibrium is a bundle* (P_1^*, P_2^*, α^*) *that satisfies the following conditions:*

$$\alpha^* = a(P_1^*, P_2^*), \tag{12}$$

$$P_1^* = P_1(\alpha^*; \theta), \tag{13}$$

$$P_2^* = P_2(1 - \alpha^*). \tag{14}$$

Condition (12) states that any equilibrium budget share α^* is consistent with consumer behavior as summarized by the budget-share mapping $a(\cdot)$ and $1 - a(\cdot)$. Conditions (13) and (14) state that the equilibrium price indices are the intrasectoral price indices evaluated at the equilibrium budget shares. To alleviate notation, let

$$\mathrm{RHS}(\alpha; \theta) \equiv a(P_1(\alpha; \theta), P_2(1 - \alpha)). \tag{15}$$

We can now show that an equilibrium exists.

Proposition 1 (Existence of equilibrium)**.** When (**A1**)–(**A2**) hold, then: (i) an equilibrium as defined by (12)–(14), exists; and (ii) there is a one-to-one correspondence between the set of equilibria and the set of solutions to the fixed point condition

$$\alpha^* = \mathrm{RHS}(\alpha^*; \theta). \tag{16}$$

Proof. We proceed in reverse order. First, part (ii) is obtained by substituting P_1^* and P_2^*, given by (13)–(14), into (12). To prove part (i), observe that for any shock θ, the mapping $\mathrm{RHS}(\cdot; \theta)$ is continuous and maps $a(P_1, P_2)$ into itself (recall that the price indices are continuous in their arguments). Hence, it has at least one fixed point by Brouwer's Theorem, i.e., (16) has at least one solution α^*. Plugging α^* into (13) and (14) yields P_1^* and P_2^*. Finally, condition (12) is satisfied by the definition of $\mathrm{RHS}(\cdot; \theta)$ in (15) and because α^* solves (16). □

Before proceeding with our analysis, we establish the following lemma.

Lemma 2 (Inter-sectoral elasticity)**.** Let $a(P_1, P_2)$ be the budget share of good 1, and let $\gamma(P_1, P_2)$ be the elasticity of substitution between goods 1 and 2. Then, the following identity holds:

$$\gamma(P_1, P_2) = 1 - \left[\mathcal{E}_{P_1}(a) + \mathcal{E}_{P_2}(1 - a)\right]. \tag{17}$$

Proof. See Appendix B. □

We now derive results of an increase in θ for welfare.

3.2 Welfare changes

How the equilibrium budget share α^* changes with θ depends on the properties of $\mathrm{RHS}(\alpha, \theta)$ as given by (16). The latter depends on whether the two goods are gross substitutes or complements. Recall that two goods are called gross complements (resp., gross substitutes) if an increase in the price for one good leads to a decrease (resp., an increase) in the demand for the other good. Since X_s is the consumption aggregator in the case of homothetic preferences, the goods are gross complements at equilibrium if and only if $\partial X_s^*/\partial P_r > 0$, $s, r = 1, 2$ and $s \neq r$. Otherwise, they are gross substitutes. These properties can be reformulated in terms of the budget-share elasticities as follows:

$$\text{gross complements:} \quad \mathcal{E}_{P_1}(\alpha) > 0, \quad \text{and} \quad \mathcal{E}_{P_2}(1-\alpha) > 0; \tag{18}$$

$$\text{gross substitutes:} \quad \mathcal{E}_{P_1}(\alpha) < 0, \quad \text{and} \quad \mathcal{E}_{P_2}(1-\alpha) < 0. \tag{19}$$

As is well known—and as seen from (17)—gross complementarity implies $\gamma(P_1, P_2) < 1$, whereas gross substitutability implies $\gamma(P_1, P_2) > 1$.

Lemma 3 (Comparative statics)**.** Assume that **(A1)**–**(A2)** hold. Then: (i) $\mathrm{RHS}(\cdot, \theta)$ is downward sloping (resp., upward sloping) if and only if the goods produced by the two sectors are gross complements (resp., gross substitutes); and (ii) an increase in θ leads to a downward (resp., an upward) shift of the RHS-locus if and only if the goods produced by the two sectors are gross complements (resp., gross substitutes).

Proof. See Appendix C. □

Lemma 3 has two important implications. First, when goods are gross complements, the equilibrium is unique and interior because α^* is downward-slopping. With gross substitutes, the equilibrium is still unique provided that the RHS-locus is not too steep. For the equilibrium to be unique and interior, the RHS-locus must intersect the $45°$-line 'from above' and never 'from below'. This holds when

$$\left.\frac{\partial \mathrm{RHS}(\alpha, \theta)}{\partial \alpha}\right|_{\alpha=\alpha^*} = \mathcal{E}_{\alpha}(P_1)\mathcal{E}_{P_1}(a) + \mathcal{E}_{1-\alpha}(P_2)\mathcal{E}_{P_2}(1-a) < 1$$

at any equilibrium.[13] We assume this to hold in what follows and refer to it as assumption **(A3)**. In what follows, we focus on interior equilibrium.[14] Doing so guarantees that the equilibrium is unique and, more importantly, that both sectors are active, which is required for thinking

[13] Formally, the expressions $\partial \mathrm{RHS}(\alpha^*, \theta)/\partial\alpha$ and $\mathcal{E}_{\alpha}(P_1)\mathcal{E}_{P_1}(a) + \mathcal{E}_{1-\alpha}(P_2)\mathcal{E}_{P_2}(1-a)$ are only well defined when α^* is an interior equilibrium. To evaluate them at a corner equilibrium, $\alpha^* = 0$ or $\alpha^* = 1$, one must replace the derivatives with appropriate one-sided derivatives.

[14] Though, we provide an example for corner equilibria in Section 4.1.1.

about intersectoral reallocations in our setting.[15] Second, gross substitutability or gross complementarity fully determines the direction in which an increase in θ shifts the budget structure: a larger share of income is spent on good 1 under gross substitutes and a smaller one under gross complements.

We can now provide a sharper answer to the central question of the paper: when do the interactions between sectors amplify or dampen the positive effect from an increase in θ? As suggested by the comparative static results, the answer depends on whether the two goods are gross substitutes or complements. It also depends on the ranking of the sectoral elasticities of the price indices with respect to the budget shares as shown by Table 1.

Table 1: Magnified or dampened shock

	Gross complements	Gross substitutes
$\lvert\mathcal{E}_\alpha(P_1)\rvert > \lvert\mathcal{E}_{1-\alpha}(P_2)\rvert$	Dampened	Amplified
$\lvert\mathcal{E}_\alpha(P_1)\rvert < \lvert\mathcal{E}_{1-\alpha}(P_2)\rvert$	Amplified	Dampened

The intuition for these results is as follows. Consider the case of gross substitutes, with the price index of sector 1 being more sensitive to changes in its budget share than that of sector 2. An increase in θ makes good 1 cheaper. Consequently, consumers spend relatively more on good 1 than on good 2, which amplifies the direct positive effect $\partial P_1/\partial\theta$ of the sector-specific shock. Although the budget reallocation reduces welfare from sector 2—since the price index increases—the positive welfare effect of the budget share reallocation to sector 1 dominates. Total welfare thus increases even more than with fixed budget shares. The intuition behind the other cases is similar.

When there is a dampening effect, a second question arises naturally: can the direct welfare-improving effect in sector 1 of an increase in θ be dampenend to the point where there are welfare losses? To answer this question, let $E(P_1, P_2, U)$ be the expenditure function associated with the upper-tier utility $U(\cdot)$. Welfare gains due to an increase in θ are then equivalent to a decrease in the equilibrium value E^* of the expenditure function in response to changes $\mathrm{d}P_s^*$ of the sectoral price indices. We can show that there exists a suffience statistic that summarizes the welfare changes and which depends solely on the elasticity of the price indices with respect to the sectoral budget shares.

Proposition 2 (Welfare multiplier)**.** Assume that **(A1)–(A3)** hold. Then the change in the expenditure function is

$$\frac{\mathrm{d}E^*}{\mathrm{d}\theta} = \frac{\alpha^*}{P_1^*}\frac{\partial P_1}{\partial\theta}\cdot\mathcal{M}, \tag{20}$$

[15] There are two additional justifications for focusing on interior equilibria. First, it leaves no room for discussions that may arise under multiple equilibria, qualifying those with counter-intuitive welfare properties as 'implausible' or 'abnormal' cases. Second, with more than two industries (see Section 5), the analysis of corner equilibria becomes a hard combinatorial problem as there are a very large number of possible cases.

where $\mathcal{M}$ is a *welfare multiplier* given by

$$\mathcal{M} = 1 + \frac{\left[\mathcal{E}_\alpha(P_1) - \mathcal{E}_{1-\alpha}(P_2)\right]\mathcal{E}_{P_1}(a)}{1 - \left[\mathcal{E}_{1-\alpha}(P_2)\mathcal{E}_{P_2}(1-a) + \mathcal{E}_\alpha(P_1)\mathcal{E}_{P_1}(a)\right]}. \tag{21}$$

Proof. See Appendix D. □

Observe that the sign of the numerator $\left[\mathcal{E}_\alpha(P_1) - \mathcal{E}_{1-\alpha}(P_2)\right]\mathcal{E}_{P_1}(a)$ of the second term of (21) depends on whether goods are gross substitutes or complements. Since the denominator is positive at any stable interior equilibrium by **(A3)**, it determines the magnifying or dampening nature of the effect. When goods are gross complements there are always magnified welfare gains. When goods are gross substitutes, the welfare gains are dampened and may even be reversed—i.e., there are welfare losses—when $1 - \left[\mathcal{E}_{1-\alpha}(P_2)\mathcal{E}_{P_2}(1-a) + \mathcal{E}_\alpha(P_1)\mathcal{E}_{P_1}(a)\right]$ is small enough. To summarize:

Corollary 1 (Welfare with gross complements)**.** When the two goods are gross complements: (i) the equilibrium is unique and interior; (ii) a higher θ always yields welfare gains, i.e., $\mathcal{M} > 0$.

Proof. Part (i) has already been shown before. To prove part (ii), restate (21) as follows:

$$\mathcal{M} = \frac{1 + \mathcal{E}_{1-\alpha}(P_2)\left[\gamma(P_1, P_2) - 1\right]}{1 - \left[\mathcal{E}_{1-\alpha}(P_2)\mathcal{E}_{P_2}(1-a) + \mathcal{E}_\alpha(P_1)\mathcal{E}_{P_1}(a)\right]}, \tag{22}$$

where we have used (17). The denominator in (22) is positive by **(A3)**. Furthermore, using **(A2)** and $\gamma(P_1, P_2) < 1$, the numerator in (22) is also positive, whence $\mathcal{M} > 0$. Therefore, (20) implies $\mathrm{d}E^*/\mathrm{d}\theta > 0$, which completes the proof. □

Corollary 2 (Welfare with gross substitutes)**.** When the two goods are gross substitutes: (i) the equilibrium is unique and interior; (ii) an increase in θ leads to a welfare loss in the vicinity of the equilibrium (P_1^*, P_2^*, α^*) when

$$\gamma(P_1^*, P_2^*) > 1 - \frac{1}{\mathcal{E}_{1-\alpha}(P_2)}, \tag{23}$$

and leads to welfare gains otherwise.

Proof. Assumption **(A3)** guarantees a unique interior equilibrium where the denominator in (22) is positive. Thus, (23) is equivalent to $\mathcal{M} < 0$, i.e., welfare losses as implied by (20). □

The intuition behind the first corollary is that, although the direct effect of the shock may be dampened by intersectoral spillovers, under gross complements this dampening is never sufficiently strong to turn gains into losses. By contrast, such a situation may occur with gross substitutes, as shown by the second corollary.

One may be worried that the case of welfare losses due to shocks that are a priori beneficial at the sectoral level is a 'pathological' case with 'zero measure'. This is not so. Indeed, any budget-share function that satisfies (23) suffices to generate welfare losses. Simple, analytically solvable, examples are provided in the next section, where we illustrate our general results for a variety of cases from the literature.

Observe that while the results in Corollaries 1 and 2 appear to be local, they have a global counterpart in the sense that they hold for large shocks provided we remain at an interior equilibrium (which is unique by **A3**). In the case of gross substitutes, this requires that condition (23) holds everywhere as long as both sectors are active. The same applies to all the comparative statics results in the rest of the paper.

4 Two industries: Applications

The welfare results from the foregoing section require that sectoral prices fall with sectoral budget shares. This property—which we take as a primitive—is generally the joint outcome of preferences and market structure and can be generated in multiple ways. We now develop four examples. We start with CES preferences and monopolistic competition to show our results in Krugman-Melitz type economies with trade or heterogeneous firms where markups are constant but price indices fall because of entry. We then: (i) switch to non-CES preferences but keep monopolistic competition; and (ii) change market structure to oligpolistic competition but keep CES preferences to show how variable markups modify our results.

4.1 Constant markups

Assume that goods 1 and 2 are gross substitutes and that X_1 and X_2 are CES aggregates with elasticities of substitution $\sigma_1 > 1$ and $\sigma_2 > 1$. Firms are identical in each sector, i.e., they have the same fixed labor requirements f_i with $i = 1, 2$ and marginal labor requirements, c_1/θ in sector 1 and c_2 in sector 2. Thus, an increase in θ leads to productivity improvements in sector 1. With monopolistic competition, the sectoral price indices are given by

$$P_1(\alpha, \theta) = K_1 \theta^{\frac{\sigma_1}{1-\sigma_1}} \alpha^{\frac{1}{1-\sigma_1}}, \qquad P_2(1-\alpha) = K_2 (1-\alpha)^{\frac{1}{1-\sigma_2}}, \tag{24}$$

where K_1, K_2 are positive constants.[16] Note that (24) immediately implies that **(A1)–(A2)** hold because $\sigma_1 > 1$ and $\sigma_2 > 1$. Since $\mathcal{E}_\alpha(P_1) = 1/(1-\sigma_1)$ and $\mathcal{E}_{1-\alpha}(P_2) = 1/(1-\sigma_2)$, (21) becomes

$$\mathcal{M} = 1 + \frac{\frac{\sigma_1 - \sigma_2}{(\sigma_1 - 1)(\sigma_2 - 1)}\mathcal{E}_{P_1}(a)}{1 + \frac{1}{\sigma_2 - 1}\mathcal{E}_{P_2}(1-a) + \frac{1}{\sigma_1 - 1}\mathcal{E}_{P_1}(a)}. \tag{25}$$

Expression (25) highlights two important results. First, $\mathcal{M} > 1$ if and only if $\sigma_1 < \sigma_2$ (recall that $\mathcal{E}_{P_1}(a) < 0$ by **(A2)** and that the denominator is positive by **(A3)**). Hence, a positive productivity shock gets amplified only if it hits the sector that has the more elastic price index with respect to the sectoral budget share. The intuition is that the positive shock makes prices fall in the more elastic sector, and this gets amplified by the subsequent budget reallocation (which decreases prices more in the elastic sector than it increases them in the less elastic sector).

Second, $\mathcal{M} = 1$ (no amplification or dampening) when either: (i) the upper-tier utility is Cobb-Douglas ($\mathcal{E}_{P_1}(a) = \mathcal{E}_{P_2}(1-a) \equiv 0$); or (ii) the elasticities of substitution are identical in both sectors ($\sigma_1 = \sigma_2$). Although both of these are knife-edge cases, they have been widely used in the literature.

We can further show that welfare losses with CES lower-tier utilities and monopolistic competition can never arise when goods 1 and 2 are poorer substitutes than varieties within sectors.

Proposition 3 (Welfare with CES)**.** Assume that goods 1 and 2 are gross substitutes and that the sectoral price indices are given by (24). Then, welfare losses occurs if and only if $\sigma_1 > \sigma_2$ and $\gamma(P_1^*, P_2^*) > \sigma_2$, i.e., the upper-tier elasticity exceeds the elasticity within sector 2.

Proof. Substituting $\mathcal{E}_{1-\alpha}(P_2) = 1/(1-\sigma_2)$ into (23), we directly obtain $\gamma > \sigma_2$. By Corollary 2, we thus have welfare losses in the vicinity of an interior equilibrium. □

Proposition 3 shows that, with CES lower-tier preferences and monopolistic competition, losses can only arise in the empirically implausible case where goods 1 and 2 are closer substitutes than the varieties in at least one of the sectors. The intuition is that firms move from sector 2 to sector 1 following a productivity improving shock in sector 1. With product differentiation, producers do not internalize the negative effect their reallocation creates—consumers value variety in sector 2 more strongly than in sector 1—which may lead to welfare losses.[17]

Note that the above analysis remains valid for a shock that affects fixed production costs instead of marginal costs. To see this, assume that the fixed labor requirement in sector 1 is f_1/θ. Then, although prices in sector 1 are independent of θ, price indices are given by

[16]Budget constraint in sector 1 implies $N_1 = \alpha/(p_1 q_1)$, where q_1 is firm output in that sector. Since $p_1 = (\sigma_1 c_1)/(\theta(\sigma_1 - 1))$ and free entry requires that $q_1 = f_1(\sigma_1 - 1)/c_1$, we obtain $N_1 = \alpha\theta/(\sigma_1 f_1)$. Hence, $P_1 = K_1 \alpha^{\frac{1}{1-\sigma_1}} \theta^{\frac{\sigma_1}{1-\sigma_1}}$, where $K_1 = \sigma_1^{\frac{\sigma_1}{\sigma_1 - 1}} c_1 f_1^{\frac{1}{\sigma_1 - 1}}/(\sigma_1 - 1) > 0$. By analogy, $K_2 = \sigma_2^{\frac{\sigma_2}{\sigma_2 - 1}} c_2 f_2^{\frac{1}{\sigma_2 - 1}}/(\sigma_2 - 1) > 0$.

[17]This is an illustration of a standard market failure where product selection is not optimal (see, e.g., Dixit and Norman, 1980, Ch.9). See Matsuyama (1995) for a discussion of that assumption and possible interpretations.

$$P_1(\alpha,\,\theta) = K_1 \theta^{\frac{1}{1-\sigma_1}} \alpha^{\frac{1}{1-\sigma_1}}, \qquad P_2(1-\alpha) = K_2 (1-\alpha)^{\frac{1}{1-\sigma_2}},$$

because the mass of firms in sector 1 is still given by $N_1 = \alpha\theta/(\sigma_1 f_1)$ and proportional to θ. As a result, Proposition 3 also applies to a decline in the fixed cost of production in sector 1.

The sectoral price indices (24) apply to CES models of trade and firm heterogeneity à la Melitz (2003). We now develop one example of each to highlight the possible origins of the productivity shock θ and show that our results hold in more fully specified models.

4.1.1 Trade liberalization

We study trade between two countries that are symmetric in terms of preferences, production costs, and population sizes. Hence, wages are equalized in equilibrium.[18] Firms in sector i have constant fixed- and marginal labor requirements, f_i and c_i with $i = 1, 2$. Trade costs between the two countries are of the iceberg form: $\tau_i \geq 1$ units of good i have to be dispatched for one unit to arrive.

By symmetry, in sector 1, there is the same mass of firms, N_1, in each country and, therefore, the same price index, P_1. Profit maximization implies that domestic and imported prices are given by $p_1^d = \sigma_1 c_1/(\sigma_1 - 1)$ and $p_1^x = \tau_1 p_1^d$, respectively. Using the profit-maximizing prices, the free entry condition takes the form $q_1 = f_1(\sigma_1 - 1)/c_1$, where q_1 is firm output in sector 1. Using the budget constraint, we obtain $\alpha = N_1 p_1^d q_1$, so that $N_1 = \alpha/(\sigma_1 f_1)$. The price index thus equals $P_1^{1-\sigma_1} = N_1 (p_1^d)^{1-\sigma_1}(1 + \tau_1^{1-\sigma_1})$. Plugging the mass of firms and the profit-maximizing price into this expression yields:

$$P_1 = K_1 \alpha^{\frac{1}{1-\sigma_1}} (1 + \tau_1^{1-\sigma_1})^{\frac{1}{1-\sigma_1}}, \quad \text{and} \quad P_2 = K_2 (1-\alpha)^{\frac{1}{1-\sigma_2}} (1 + \tau_2^{1-\sigma_2})^{\frac{1}{1-\sigma_2}}, \tag{26}$$

where the price index P_2 in sector 2 is obtained in the same way than for sector 1, and where K_1 and K_2 are positive constants.[19] In this subsection, we consider that our sectoral shock amounts to trade liberalization, i.e., we switch from τ_1 to τ_1/θ, where $\theta \geq 1$ represents the decrease in trade costs in sector 1.

Corollary 1 shows that when the goods produced by the two sectors are gross complements, trade liberalization in sector 1, always yields welfare gains. However, when the goods are gross substitutes, trade liberalization in sector 1 may lead to welfare losses even with well-behaved upper-tier preferences. Losses from trade arise when love-of-variety for the good produced by sector 1—where trade liberalization takes place—is lower than love-of-variety for the good produced by sector 2 and the sectoral price indices are sufficiently elastic with respect to sectoral budget shares. The intuition is that trade liberalization in sector 1 results in a higher budget share

[18] Since we are looking at a symmetry trade shock, wages remain equalized after the shock.

[19] They are given by $K_1 = \sigma_1^{\frac{\sigma_1}{\sigma_1 - 1}} f_1^{\frac{1}{\sigma_1 - 1}} c_1/(\sigma_1 - 1)$ and $K_2 = \sigma_2^{\frac{\sigma_2}{\sigma_2 - 1}} f_2^{\frac{2}{\sigma_2 - 1}} c_2/(\sigma_2 - 1)$.

α for this sector, which reduces product diversity in sector 2. As a result, even though product diversity increases in the first sector, consumers may end up worse when the losses in sector 2 outweight the gains in sector 1. Hence, lower prices due to lower costs do not necessarily make consumers better off in a multi-sector settings with free entry.

To show that losses from trade are not a zero measure case, consider the CES upper-tier utility function (2), which implies the following budget share function

$$a(P_1, P_2) = \frac{P_1^{1-\gamma}}{P_1^{1-\gamma} + P_2^{1-\gamma}}, \tag{27}$$

where $\gamma > 1$ stands for the intersectoral elasticity of substitution. Since

$$\mathcal{E}_{P_1}(a) = -(\gamma - 1)(1 - a) < 0, \quad \text{and} \quad \mathcal{E}_{P_2}(1 - a) = -(\gamma - 1)a < 0, \tag{28}$$

it corresponds to the case of gross substitutes. The elasticities of the price indices to the budget shares are still given by $\mathcal{E}_\alpha(P_1) = 1/(1 - \sigma_1)$, and $\mathcal{E}_{1-\alpha}(P_2) = 1/(1 - \sigma_2)$ as shown by (26).

Plugging (27) and (28) into (25) and simplifying, we get

$$\mathcal{M} = \frac{(\sigma_2 - \gamma)(\sigma_1 - 1)}{(\sigma_1 - 1)(\sigma_2 - 1) - (\gamma - 1)\left[\alpha(\sigma_1 - \sigma_2) + \sigma_2 - 1\right]} \tag{29}$$

Since the denominator is positive by (**A3**), losses from trade arise under $\gamma > \sigma_2$ as stated in Proposition 3.

We now illustrate this with the help of two different parametrizations, where we assume, without loss of generality, that $1 < \sigma_2 < \sigma_1$. To this end, substituting the price indices into (27) and rearranging, we obtain the following fixed point condition:

$$a(P_1, P_2) = \frac{1}{1 + K_3(1 + \tau_1^{1-\sigma_1})^{\frac{\gamma-1}{1-\sigma_1}} \alpha^{\frac{\gamma-1}{1-\sigma_1}} (1 - \alpha)^{\frac{\gamma-1}{\sigma_2-1}}}, \tag{30}$$

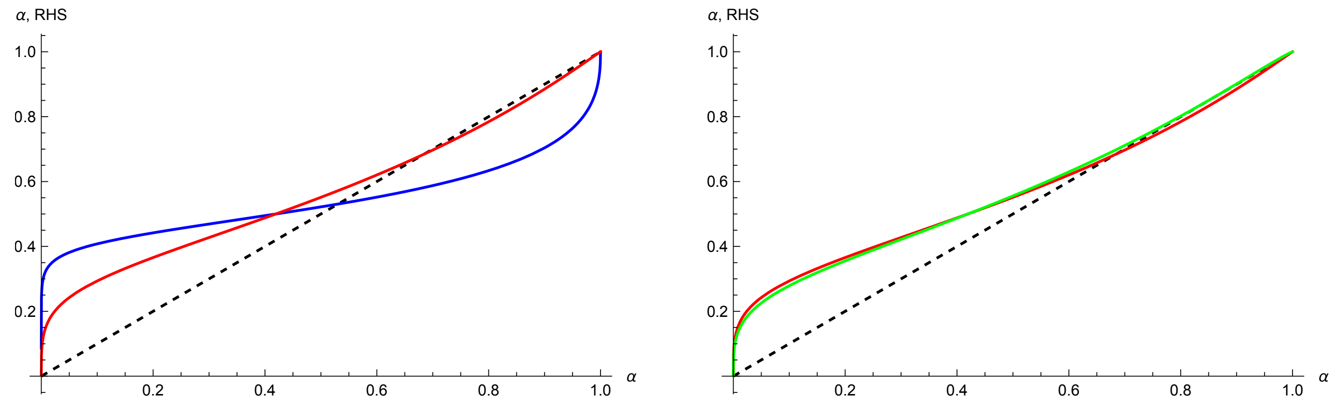
where K_3 is a positive constant.[20] Observe that the RHS is increasing in α^* with $\lim_{\alpha^* \to 0} RHS = 0$ and $\lim_{\alpha^* \to 1} RHS = 1$.

In our examples in panel (a) of Figure 1, we have $\alpha^* = 0.531$ for the blue line, with a corresponding $\mathcal{M} = 0.794$. Hence, a positive trade shock to sector 1 increases welfare, though the gains are dampened by the presence of the substitute sector 2. For the red line, $\alpha^* = 0.685$ with a corresponding $\mathcal{M} = -0.156$. Hence, this case corresponds to one where a positive trade shock in sector 1 would reduce welfare in the economy (since $\gamma > \sigma_2 = 4.2$, above which we get welfare losses following the positive shock). The reason is that goods in sector 1 are relatively less differentiated than goods in sector 2, but the price index in sector 1 is relatively elastic with respect to

[20] It is given by $K_3 = \left[K_1 / \left(K_2(1 + \tau_2^{1-\sigma_2})^{\frac{1}{1-\sigma_2}}\right)\right]^{\gamma-1}$

Figure 1: Equilibrium budget shares with CES upper-tier preferences, international trade.

(a) Baseline equilibrium (b) Switch to corner solution



Notes: The parameter values are $K_1 = K_2 = 1$, $\sigma_1 = 12.1$, $\sigma_2 = 4.2$, $\tau_1 = 2.5$, $\tau_2 = 1.4$, and $\theta = 1$. In panel (a), the blue solid curve sets $\gamma = 2.4$, whereas the red curve depicts the case with $\gamma = 4.3$. In panel (b), the red solid curve sets $\gamma = 2.4$, whereas the red dashed curve depicts the case with $\gamma = 4.556$.

the sectoral budget shares. Hence, a positive shock to sector 1 leads to a substantial reallocation of budget to sector 1, which reduces variety in sector 2. The latter being more strongly valued, welfare falls.[21] Panel (b) shows that further increases in γ lead to a sudden switch to a corner equilibrium, where sector 2 completely shuts down and $\alpha^* = 1$. The happens for $\gamma = 4.356$, where the budget share jumps from around 0.86 to 1.[22] At the corner equilibrium, the positive shock to trade costs in sector 1 then obviously raise welfare since only the direct effect of the shock materializes and

$$\mathcal{M} \Big|_{\alpha^*=1} = \frac{(\sigma_2 - \gamma)(\sigma_1 - 1)}{(\sigma_1 - 1)(\sigma_2 - 1) - (\gamma - 1)\left((\sigma_1 - \sigma_2) + \sigma_2 - 1\right)} = 1,$$

as expected. This shows that the effects of positive shocks on welfare depend on the type of equilibrium and the value of γ, and may change discontinuously.

4.1.2 Productivity improvements

Consider next a closed two-sector economy where both sectors are monopolistically competitive and firms are heterogeneous in productivity à la Melitz (2003). As before, the lower-tier utilities are CES, with elasticities of substitution $1 < \sigma_2 < \sigma_1$. Let $\Gamma_0(\cdot)$ and $\Gamma_1(\cdot)$ be two cumulative distribution functions (CDF) of firms' marginal costs—the inverse of productivity—defined over $\mathbb{R}_+$ such that $\Gamma_0(\cdot)$ first-order stochastically dominates $\Gamma_1(\cdot)$. Assume that the CDF of marginal

[21] This argument is reminiscent of the "trade in cultural goods" case (see, e.g., Francois and van Ypersele, 2002), where less valued foreign varieties displace more valued domestic ones. In our case, there are no foreign or domestic varieties in a strict sense, but the market failure is still the displacement of more preferred varieties by less preferred ones following the trade shock.

[22] This is an illustration of the market failures highlighted in Matsuyama (1995).

costs c in sector 1 is $\Gamma_\theta(c) \equiv (1-\theta)\Gamma_0(c) + \theta\Gamma_1(c),\ \theta \in [0,\,1]$, whereas that in sector 2 is $\Gamma_1(c)$. Hence, a decrease in θ corresponds to a left-shift of the cost distribution in sector 1, i.e., a positive productivity shock.

Following Melitz (2003), the cutoff cost $\bar{c}_1$ and the cutoff firm output $\bar{q}_1$ are uniquely determined by the zero cutoff profit condition

$$\bar{q}_1 = \frac{(\sigma_1 - 1)f_1}{\bar{c}_1} \tag{31}$$

and the zero expected profit condition

$$\int_0^{\bar{c}_1} \left[\left(\frac{\bar{c}_1}{c}\right)^{\sigma_1 - 1} - 1\right] \mathrm{d}\Gamma_\theta(c) = \frac{f_e}{f_1}, \tag{32}$$

where f_1 is the fixed cost of production and f_e the sunk entry cost for a firm. Because the left-hand side of (32) is an increasing function of $\bar{c}_1$, (32) has a unique solution $\bar{c}_1 = \bar{c}_1(\theta)$. Furthermore, using the expression for $\Gamma_\theta(\cdot)$ and (32) yields that $\bar{c}_1(\theta)$ increases in θ. Because the lower-tier utilities are CES, the market clearing condition and the optimal pricing rule imply the following equalities: $\bar{q}_1 = \frac{\alpha}{P_1}\left(\bar{p}_1/P_1\right)^{-\sigma_1}$ and $\bar{p}_1 = \frac{\sigma_1}{\sigma_1 - 1}\bar{c}_1$, where $\bar{p}_1$ is the price of the cutoff firm. Plugging these expressions into (31) and solving the resulting equation for P_1, we obtain

$$P_1(\theta, \alpha) = K_1 \bar{c}_1(\theta)\alpha^{\frac{1}{1-\sigma_1}}, \tag{33}$$

where $K_1 \equiv \frac{\sigma_1}{\sigma_1 - 1}\left(\sigma_1 f_1\right)^{1/(\sigma_1 - 1)} > 0$ is a positive bundle of parameters that depends neither on α nor on θ. The corresponding expression for sector 2 is

$$P_2(1-\alpha) = K_2 \bar{c}_2 (1-\alpha)^{\frac{1}{1-\sigma_2}},$$

where $K_2 \equiv \frac{\sigma_2}{\sigma_2 - 1}\left(\sigma_2 f_2\right)^{1/(\sigma_2 - 1)} > 0$, which does not depend on θ since $\Gamma_2(\cdot)$ is independent of it. Given the foregoing expressions, Proposition 3 applies: there exist well-behaved upper-tier utility functions exhibiting gross substitutability such that a decrease in θ—an improvement in sector 1 productivity—leads to welfare losses.

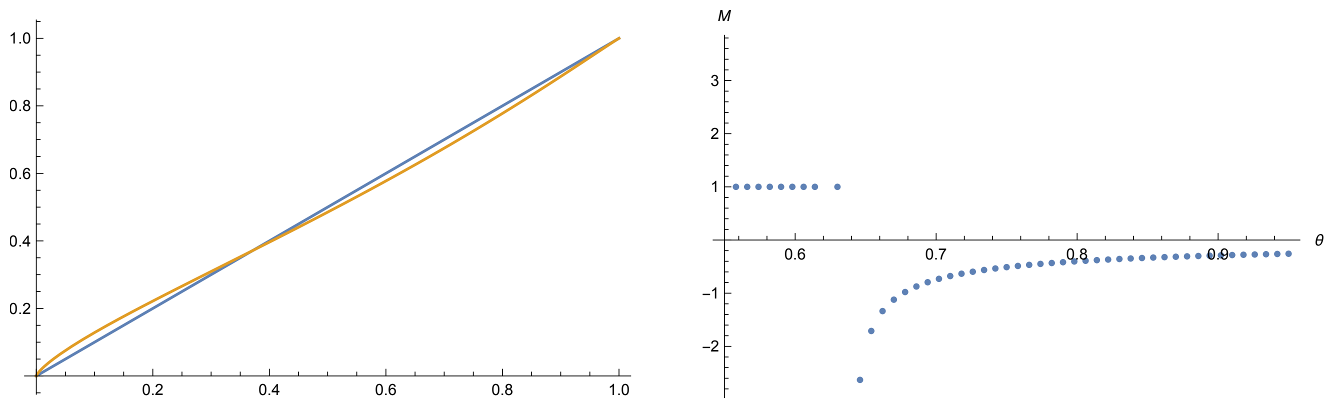
To see this, we provide a numerical example. Assume again that the budget share function is (27), with elasticities (28). Furthermore, $\mathcal{E}_\alpha(P_1) = \frac{1}{1-\sigma_1} < 0$ and $\mathcal{E}_{1-\alpha}(P_2) = \frac{1}{1-\sigma_2} < 0$ continue to hold. Since $\sigma_1 > \sigma_2$, $\mathcal{E}_\alpha(P_1) < \mathcal{E}_{1-\alpha}(P_2)$. Hence, from (25), at any unique interior equilibrium $\mathcal{M} < 1$, i.e., part of the gains in sector 1 are offset by losses in sector 2. We can readily verify that $\mathcal{M} < 0$ can occur, i.e., the gains in sector 1 are more than offset by the losses in sector 2. To see this, let

$$\Gamma_1(c) = (1-\theta)\log\mathcal{N}(2,1) + \theta\log\mathcal{N}(3,1), \quad \text{and} \quad \Gamma_2(c) = \log\mathcal{N}(3,1), \tag{34}$$

where $\log \mathcal{N}(\mu, \sigma)$ is the lognormal distribution with mean μ and standard deviation σ.[23] Starting with $\theta = 1$, both sectors have the same underlying productivity distribution. As θ decreases, sector 1 progressively becomes more productive.

Figure 2: Equilibrium budget shares with CES upper-tier preferences, heterogeneous firms.

(a) Baseline equilibrium (b) Welfare multiplier $\mathcal{M}$



Notes: The parameter values are $f_1 = f_2 = 1$, $f_e = 0.1$, $\sigma_1 = 1.4$, $\sigma_2 = 1.3$, $\tau_1 = 2.5$, $\tau_2 = 1.4$, and $\gamma = 1.31$. In panel (a), we let $\theta = 0.95$. Panel (b) depicts the welfare multiplier as a function of θ.

Panel (a) in Figure 2 shows that there is a stable interior equilibrium around $\alpha^* = 0.38$. Panel (b) shows that there are welfare losses from a positive productivity shock at that equilibrium since the welfare multiplier $\mathcal{M}$ is negative. As θ decreases, the negative effects of the shock become stronger until we jump back to $\mathcal{M} = 1$ once sector 2 shuts down, as explained before in the trade section.

4.2 Variable markups

CES monopolistic competition models channel welfare effects through changes in product diversity, with reduced diversity in one sector responsible for the dampening of welfare changes. However, a second negative effect also arises in non-CES models: an increase in prices due to a smaller market and less competition, which raises markups. When markups are variable in sector 2, welfare losses may occur even when $\gamma < \sigma_2$ provided that prices are 'elastic enough'. There are at least two ways in which markups can be made variable: (i) if we depart from CES preferences in sector 2 and have increasing elasticity of demand; or (ii) if we depart from monopolistic competition in sector 2 and have, e.g., oligopolistic competition. In those cases, a shrinking market for sector 2 leads to both less product diversity and higher prices in this sector. The combination of these two negative general equilibrium effects can dominate the positive

[23]One may think that using Pareto distributions with the same shape parameter but with different supports—where the support of sector 1 is parametrized by θ—would simplify the analysis. We can indeed solve for the cutoffs in a closed form, but they depend in a highly non-linear way on the elasticities σ_1 and σ_2. We can thus not get simple results, even with Pareto distributions.

direct effect of a shock to sector 1.

We develop two examples to illustrate this. In both examples, we assume that the two goods are gross substitutes and that there is a productivity-enhancing shock in sector 1. In the first example, we assume non-CES preferences in sector 2, which gives rise to pro-competitive effects. In the second example, we maintain CES preferences in both sectors but assume the market structure in sector 2 is oligopolistically competitive. In both cases, welfare losses can arise even when the inter-sectoral elasticity of substitution is smaller than the elasticity of substitution in both sectors.

4.2.1 Non-CES monopolistic competition.

We assume homothetic non-CES preferences in sector 2 following Kimball (1995). There exists an increasing and concave function ψ such that utility X_2 in sector 2 is implicitly defined as

$$\int_0^{N_2} \psi\left(\frac{x_2(i)}{X_2}\right) \mathrm{d}i = 1, \tag{35}$$

where N_2 is the mass of firms in sector 2. The budget constraint is $\int_0^{N_2} p_2(i)x_2(i)\mathrm{d}i = 1-\alpha$, where $p_2(i)$ denotes the price of variety i. The first-order conditions in sector 2 are given by

$$p_2(i) = \frac{\psi'(z(i))}{\mu}, \tag{36}$$

where $z(i) = x_2(i)/X_2$ and where μ is a market aggregate that involves the Lagrange multiplier of the budget constraint.

Each firms incurs a fixed cost, f_2, and a constant marginal cost, c_2, both paid in terms of labor. The profit of firm i in sector 2 is $\pi_2(i) = (p_2(i) - c_2)Lx_2(i) - f_2$. Maximizing it with respect to quantities and using the inverse demands (36), we obtain

$$p_2 = \frac{c_2}{1 - r_\psi(z)}, \quad \text{where} \quad r_\psi(z) = -\frac{z\psi''(z)}{\psi'(z)} \tag{37}$$

is related to the markup.

In what follows, we focus on symmetric equilibria. Equation (35) implies $N_2\psi(z) = 1$ the budget constraint implies $N_2 p_2 x_2 = 1-\alpha$, and homotheticity of preferences implies $P_2X_2 = 1-\alpha$. Combining these conditions implies that the price index is given by

$$P_2 = \frac{1}{\psi(z)} \frac{c_2 z}{1 - r_\psi(z)}. \tag{38}$$

Plugging (37) into the zero-profit condition yields the zero-profit quantity $x_2 = \frac{f_2}{Lc_2}\frac{1-r_\psi(z)}{r_\psi(z)}$. Replacing x_2 in the budget constraint and recalling that $N_2 = 1/\psi(z)$, we then obtain

$$1 - \alpha = \frac{f_2}{L} \frac{1}{\psi(z) r_\psi(z)}. \tag{39}$$

Under monopolistic competition, the demand elasticity equals the elasticity of substitution. Thus, since $r_\psi(z) = \frac{\partial p}{\partial z}\frac{z}{p}$ from (36), we obtain

$$\sigma_2(z) = \frac{1}{r_\psi(z)}. \tag{40}$$

Using (39), we compute the elasticity of $1-\alpha$ with respect to z and invert it to get:

$$\mathcal{E}_{1-\alpha}(z) = -\frac{1}{z}\frac{\psi(z) r_\psi(z)}{\psi'(z) r_\psi(z) + \psi(z) r'_\psi(z)},$$

whereas (38) yields

$$\mathcal{E}_{1-\alpha}(P_2) = \left[1 - \frac{z\psi'(z)}{\psi(z)} + \frac{z r'_\psi(z)}{1 - r_\psi(z)}\right] \varepsilon_\alpha(z).$$

Combining these two equations, we obtain

$$\mathcal{E}_{1-\alpha}(P_2) = -\left[1 - \frac{z\psi'(z)}{\psi(z)} + \frac{z r'_\psi(z)}{1 - r_\psi(z)}\right] \frac{1}{\frac{z\psi'(z)}{\psi(z)} + \frac{z r'_\psi(z)}{r_\psi(z)}}. \tag{41}$$

We are now ready to compare the impact of the shock in sector 1 for CES and non-CES preferences in sector 2. To make clear-cut comparison, we use the following parametrisation for Kimball preferences:

$$\psi(z) = z^\rho - \varepsilon z, \tag{42}$$

with $\rho \in (0,1)$ and $\varepsilon > 0$. This function is increasing and concave for $z \leq \left(\frac{\varepsilon}{\rho}\right)^{\frac{1}{\rho-1}}$. It follows that

$$r_\psi(z) = \rho(1-\rho)\frac{z^{\rho-1}}{\rho z^{\rho-1} - \varepsilon} \quad \text{and} \quad r'_\psi(z) = \varepsilon\rho(1-\rho)^2 \frac{z^{\rho-2}}{(\rho z^{\rho-1} - \varepsilon)^2} > 0,$$

which shows that these preferences generate pro-competitive effects.

Under CES preferences, $\varepsilon \to 0$ and $r_\psi(z) = 1 - \rho$ so that $r'_\psi(z) = 0$, which implies that (38) reduces to

$$\mathcal{E}_{1-\alpha}(P_2) = 1 - \frac{1}{\frac{z\psi'(z)}{\psi(z)}} = 1 - \frac{1}{\rho}.$$

It then follows from (40) that $\sigma_2(z) = \frac{1}{1-\rho}$. Therefore, condition (23) in Proposition 3, namely $\gamma > 1 - \frac{1}{\mathcal{E}_{1-\alpha}(P_2)}$, converges to $\gamma > \sigma_2$ as before.

Under non-CES preferences, $\varepsilon > 0$. We can then rewrite (41) as follows

$$\mathcal{E}_{1-\alpha}(P_2) = 1 - \frac{1}{\frac{z\psi'(z)}{\psi(z)}} - \frac{zr'_\psi(z)}{r_\psi(z)} \frac{\frac{z\psi'(z)}{\psi(z)}\frac{1}{1-r_\psi(z)} - 1}{\left[\frac{z\psi'(z)}{\psi(z)} + \frac{zr'_\psi(z)}{r_\psi(z)}\right]\frac{z\psi'(z)}{\psi(z)}}.$$

Note that $\frac{z\psi'(z)}{\psi(z)}$ equals ρ in the CES case, whereas it equals $\frac{\rho z^\rho - \varepsilon z}{z^\rho - \varepsilon z} < \rho$ in the non-CES case. It follows that the term $1 - \frac{1}{\frac{z\psi'(z)}{\psi(z)}}$ is larger (in absolute value; recall that $\frac{z\psi'(z)}{\psi(z)} \in (0,1)$ by concavity) in the CES case than in the non-CES case, which suggests that the price index is more elastic to the budget share. A sufficient condition for the latter to be true is that the last term—which vanishes in the CES case since $r'_\psi(z) = 0$—is negative. Since $\psi(z)$ is an increasing function and since $r'_\psi(z) > 0$, the last term is negative if

$$\frac{z\psi'(z)}{\psi(z)}\frac{1}{1-r_\psi(z)} - 1 > 0 \quad \text{or} \quad 1 - r_\psi(z) - \frac{z\psi'(z)}{\psi(z)} < 0.$$

Under our parametrisation (42), this condition takes the form

$$-\frac{(1-\rho)^2 \varepsilon z^{\rho-1}}{(\rho z^{\rho-1} - \varepsilon)(z^{\rho-1} - \varepsilon)} < 0$$

which always holds because $\varepsilon > 0$ and $z \leq (\varepsilon/\rho)^{\frac{1}{\rho-1}}$. Therefore, for $\varepsilon > 0$, price index in sector 2 is more elastic than in case of CES, i.e., $\varepsilon_{1-\alpha}(P_2)$ is larger (in absolute value).

As a result, there exists $\overline{\gamma} > 1$ with $\overline{\gamma} < \sigma_2(z)$ such that for $\gamma \in (\overline{\gamma}, \sigma_2(z))$, (23) holds. Consequently, a positive shock in sector 1 leads to overall welfare losses. The intuition is that the price index is more elastic under non-CES preferences than under CES preferences as it combines both less variety *and* higher prices, thus leading to larger shifts in budget between sectors following the shock.

4.2.2 Oligopolistic competition

We now move away from monopolistic competition and assume oligopolistic Cournot competition in sector 2. To this end, we assume that preferences for product 2 are given by

$$X_2 = \left[\sum_{i=1}^{N_2} x_2(i)^{\frac{\sigma_2 - 1}{\sigma_2}}\right]^{\frac{\sigma_2}{\sigma_2 - 1}}, \tag{43}$$

where N_2 is the number of firms competing in sector 2, and where $\sigma_2 > \gamma > 1$ is the elasticity of substitution between varieties.

We start with consumers' utility maximization problem. Using two-stage budgeting, the first-order condition for variety i is given by:

$$\frac{X_2}{\sum_{i=1}^{N_2} x_2(i)^{\frac{\sigma_2-1}{\sigma_2}}} x_2(i)^{-\frac{1}{\sigma_2}} - \lambda_2 p_2(i) = 0,$$

where λ_2 is the marginal utility of additional spending on good 2 and $p_2(i)$ price for variety i. Multiplying by $x_2(i)$ and summing on i yields $X_2 = \lambda_2(1-\alpha)$, i.e., $\lambda_2 = X_2/(1-\alpha)$. We thus have the inverse demands

$$p_2(i) = (1-\alpha)\frac{x_2(i)^{-\frac{1}{\sigma_2}}}{X_2^{\frac{\sigma_2-1}{\sigma_2}}}. \tag{44}$$

Turning next to profit maximization, the profit of firm i is given by

$$\pi_2(i) = \left[(1-\alpha)\frac{x_2(i)^{-\frac{1}{\sigma_2}}}{X_2^{\frac{\sigma_2-1}{\sigma_2}}} - c_2\right] x_2(i) - f_2, \tag{45}$$

where $c_2 > 0$ is the constant marginal cost and f_2 is the fixed costs that we assume identical across all firms operating in sector 2. Each firm chooses the quantity $x_2(i)$ that maximizes its profit, taking as given the quantities produced by the other firms. Taking into account that—unlike in monopolistic competition—each firm has an impact on the market aggregate X_2, we have:

$$(1-\alpha)\frac{\sigma_2-1}{\sigma_2}\frac{x_2(i)^{-\frac{1}{\sigma_2}}}{X_2^{\frac{\sigma_2-1}{\sigma_2}}}\left[1-\left(\frac{x_2(i)}{X_2}\right)^{\frac{\sigma-1}{\sigma}}\right] = c_2. \tag{46}$$

Conditions (44) and (46) imply that the price set by all N_2 firms at a symmetric Cournot-Nash equilibrium (where $x_2(i) = x_2$ for all i and $X_2^{\frac{\sigma_2-1}{\sigma_2}} = N_2 x_2^{\frac{\sigma_2-1}{\sigma_2}}$) is given by

$$p_2 = c_2 \frac{\sigma_2}{\sigma_2-1}\frac{N_2}{N_2-1}. \tag{47}$$

Because of the budget constraint, at the symmetric equilibrium we have $N_2 p_2 x_2 = 1-\alpha$. Hence, the Cournot-Nash equilibrium output $x_2(N_2)$ of each firm is given by

$$x_2(N_2) = \frac{1-\alpha}{N_2 p_2} = \frac{1-\alpha}{c_2}\frac{\sigma_2-1}{\sigma_2}\frac{N_2-1}{N_2^2}. \tag{48}$$

Combining (45) with (48), we find that the equilibrium profit $\pi_2(N_2)$ of each firm in sector 2 is given by

$$\pi_2(N_2) = \frac{1-\alpha}{N_2} - c_2 x_2 - f_2 = \frac{1-\alpha}{N_2}\left(1 - \frac{\sigma_2-1}{\sigma_2}\frac{N_2-1}{N_2}\right) - f_2. \tag{49}$$

We are now equipped to determine the number N_2 of firms operating in sector 2, which is pinned down by the free-entry condition $\pi_2(N_2) = 0$. Solving this quadratic equation for N_2 yields a unique positive solution

$$N_2^* = N_2^*(\alpha) = \frac{1-\alpha}{2f_2\sigma_2}\left(1+\sqrt{1+4f_2\sigma_2\frac{\sigma_2-1}{1-\alpha}}\right), \tag{50}$$

which depends on the endogenous budget share $\alpha = a(P_1, P_2)$.

At the symmetric equilibrium, the CES price index is given by $P_2 = N_2^{\frac{1}{1-\sigma_2}} p_2$ so that the elasticity $\mathcal{E}_{1-\alpha}(P_2)$ can be expressed as

$$\mathcal{E}_{1-\alpha}(P_2) = \frac{1}{1-\sigma_2}\mathcal{E}_{1-\alpha}(N_2) + \mathcal{E}_{1-\alpha}(p_2) = \mathcal{E}_{1-\alpha}(N_2)\left[\frac{1}{1-\sigma_2} + \mathcal{E}_{N_2}(p_2)\right]. \tag{51}$$

Using (47), we have $\mathcal{E}_{N_2}(p_2) = -\frac{1}{N_2-1}$, so that

$$\mathcal{E}_{1-\alpha}(P_2) = -\left[\frac{1}{\sigma_2-1} + \frac{1}{N_2-1}\right]\mathcal{E}_{1-\alpha}(N_2). \tag{52}$$

Using (50), we further have

$$\mathcal{E}_{1-\alpha}(N_2) = \frac{N_2 + (\sigma_2-1)}{N_2 + 2(\sigma_2-1)} \in (0,1). \tag{53}$$

Plugging (53) into (52), we obtain

$$\mathcal{E}_{1-\alpha}(P_2) = -\left(\frac{1}{\sigma_2-1} + \frac{1}{N_2-1}\right)\frac{N_2 + (\sigma_2-1)}{N_2 + 2(\sigma_2-1)} < 0. \tag{54}$$

Finally, inserting (54) into the condition for welfare losses, $\gamma > 1 - 1/\mathcal{E}_{1-\alpha}(P_2)$, and solving for N_2 implies there are welfare losses for $N_2^* < \overline{N}_2$, where

$$\overline{N}_2 \equiv 1 + \left(\sigma_2 - \frac{1}{2}\right)\left[\sqrt{1 + \frac{\sigma_2(\sigma_2-1)}{(\sigma_2-1/2)^2}\frac{\gamma-1}{\sigma_2-\gamma}} - 1\right]. \tag{55}$$

In words, aggregate welfare losses arise following a positive shock to sector 1 when there are few enough firms in sector 2. This holds true when, for example, fixed costs f_2 are high since N_2 monotonically decreases with f_2 (see (50)).

The intuition for the existence of welfare losses is the following. First, when fixed costs are large only a small number of firms operates in sector 2. A positive shock to sector 1—followed by a reallocation of budget towards that sector—forces firms out of sector 2. Since there are only few firms operating in that sector, any shock that reduces the mass of firms is likely to cause a substantial increase in prices. Second, there is the variety effect. The positive shock in sector 1 triggers entry and, thus, increases welfare because consumers love variety. Yet, this is at least partly offset by the loss of variety in sector 2. As explained before, when $\sigma_1 > \sigma_2$, $\mathcal{M} < 1$ (since $\mathcal{E}_{P_1}(a) < 0$): more preferred varieties in sector 2 are displaced by less preferred varieties in sector 1. The combination of both effects may be strong enough to reduce welfare following a positive shock to sector 1.

To summarize, welfare losses are likely in economies where some industries are strongly concentrated, i.e., 'granular'. Given the recent literature that emphasizes the aspect of granularity in macroeconomics (e.g., Gabaix, 2011), industrial organization (e.g., Shimomura and Thisse, 2012), and trade (e.g., Di Giovanni *et al.*, 2014; Parenti, 2018), this seems important to keep in mind when assessing the welfare effects of trade or technology shocks.

5 Beyond two sectors

We now extend our analysis to an arbitrary number n of sectors and derive the welfare multipliers for sector-specific efficiency shocks. The aim of this section is to stress the similarity between our approach, which focuses on the demand-side-driven cross-sectoral effects channeled through the budget shares, and the input-output approach, which focuses on the supply-side-driven cross-sectoral linkages channeled through the cost shares.

In what follows, we alleviate notation by using the hat operator, $\widehat{(\cdot)}$, which relates a variable to its percentage change. Formally, $\widehat{x} \equiv \frac{dx}{x}$ for a scalar variable $x \in \mathbb{R}$, and $\widehat{\mathbf{x}} \equiv (\widehat{x}_1, \widehat{x}_2, \ldots, \widehat{x}_n)$ for a vector variable $\mathbf{x} = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$.

Let $\mathbf{P} = (P_1, P_2, \ldots, P_n)$ be the vector of sectoral price indices, and let $W = V(\mathbf{P})$ be the upper-tier indirect utility (income is again fixed and normalized to one). Let $\mathbf{a}(\mathbf{P}) \in \Delta_{n-1}$, where Δ_{n-1} is the unit simplex of $\mathbb{R}^{n-1}$, denote the vector of budget shares.

In what follows, we suppress the argument $\mathbf{P}$ in $\mathbf{a}(\mathbf{P})$ and $V(\mathbf{P})$ when there is no possible confusion.

5.1 Cross elasticity effects and sectoral equilibrium

We first derive the responses of the budget shares to the price changes. Similar to Section 3, let $\boldsymbol{\alpha} \in \Delta_{n-1}$ be the vector of equilibrium budget shares.

Proposition 4 (Budget share mapping)**.** The responses of the budget shares to changes in the price indices are given by:

$$\widehat{\boldsymbol{\alpha}} = \boldsymbol{\mathcal{E}}_{\mathbf{P}}(\mathbf{a})\widehat{\mathbf{P}}, \tag{56}$$

where $\boldsymbol{\mathcal{E}}_{\mathbf{P}}(\boldsymbol{a})$ is the $(n \times n)$-matrix of the own- and cross-price elasticities of budget shares:

$$\boldsymbol{\mathcal{E}}_{\mathbf{P}}(\mathbf{a}) \equiv \begin{pmatrix} \mathcal{E}_{P_1}(a_1) & \mathcal{E}_{P_2}(a_1) & \cdots & \mathcal{E}_{P_n}(a_1) \\ \mathcal{E}_{P_1}(a_2) & \mathcal{E}_{P_2}(a_2) & \cdots & \mathcal{E}_{P_n}(a_2) \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{E}_{P_1}(a_n) & \mathcal{E}_{P_2}(a_n) & \cdots & \mathcal{E}_{P_n}(a_n). \end{pmatrix}, \tag{57}$$

Proof. The result follows immediately from log-differentiating both sides of the equilibrium condition $\boldsymbol{\alpha} = \mathbf{a}(\mathbf{P})$. □

For each sector $i \in \{1, 2, \ldots, n\}$, we define the sectoral equilibrium as a mapping of the efficiency parameter θ_i and the market size $\alpha_i \in (0, 1)$ to the sectoral price index $P_i = g_i(\theta_i, \alpha_i)$, where g_i is a function that captures the details of the technology and the market structure in sector i.[24] By definition, the sectoral equilibrium is an equilibrium computed as if a sector was an entirely autonomous one-sector economy with a given market size α_i. In other words, we fully disregard the presence of the intersectoral linkages when computing the sectoral equilibrium. We can write the collection of sectoral equilibria in vector-matrix form as follows:

$$\mathbf{P} = \mathbf{g}(\boldsymbol{\theta}, \boldsymbol{\alpha}). \tag{58}$$

By applying the hat operator to both sides of (58), we get:

$$\widehat{\mathbf{P}} = [\mathcal{E}_{\boldsymbol{\theta}}\mathbf{g}]\,\widehat{\boldsymbol{\theta}} + [\mathcal{E}_{\boldsymbol{\alpha}}\mathbf{g}]\,\widehat{\boldsymbol{\alpha}}, \tag{59}$$

where $\mathcal{E}_{\boldsymbol{\theta}}\mathbf{g}$ and $\mathcal{E}_{\boldsymbol{\alpha}}\mathbf{g}$ are diagonal $(n \times n)$-matrices given, respectively, by

$$\mathcal{E}_{\boldsymbol{\theta}}\mathbf{g} = \operatorname{diag}\begin{pmatrix} \mathcal{E}_{\theta_1} g_1(\theta_1, \alpha_1) \\ \mathcal{E}_{\theta_2} g_2(\theta_2, \alpha_2) \\ \vdots \\ \mathcal{E}_{\theta_n} g_n(\theta_n, \alpha_n) \end{pmatrix} \quad \text{and} \quad \mathcal{E}_{\boldsymbol{\alpha}}\mathbf{g} = \operatorname{diag}\begin{pmatrix} \mathcal{E}_{\alpha_1} g_1(\theta_1, \alpha_1) \\ \mathcal{E}_{\alpha_2} g_2(\theta_2, \alpha_2) \\ \vdots \\ \mathcal{E}_{\alpha_n} g_n(\theta_n, \alpha_n) \end{pmatrix}.$$

5.2 Welfare multiplier

We now study the welfare consequences of a 'good' sectoral shock. Without loss of generality, we normalize the indirect utility (using a suitable monotone transformation) so that the elasticity of the indirect utility with respect to income is unitary. Then, the percentage change $\widehat{W}$ of the

[24] Recall that income is normalized to one, hence α_i equals the total expenditure on good i and is a measure of market size.

indirect utility—which serves as our measure of welfare change—can be written as follows:

$$\widehat{W} = -\sum_{j=1}^{n} \alpha_i \widehat{P}_i = -\boldsymbol{\alpha}^T \widehat{\mathbf{P}}. \tag{60}$$

Observe that (60) is the 'consumer analogue' of Hulten's (1978) theorem, which states that (in competitive markets) the impact on aggregate TFP of a firm-level TFP shock equals the value of the shock times the firm's sales as a share of GDP. In our case, the welfare effects of a shock to sector i equals the direct effect of the shock times the consumer's budget share spent on goods produced by that sector.

The following proposition provides a characterization of the welfare change in response to a sectoral shock through a sector-specific welfare multiplier.

Proposition 5 (Multisector welfare multiplier)**.** Assume that only sector i is affected by a welfare-improving shock $\widehat{\boldsymbol{\theta}} = \widehat{\theta}_i \mathbf{e}_i$, where $\mathbf{e}_i$ is the ith vector of the standard basis. Then, the welfare change is given by:

$$\widehat{W} = \mathcal{M}_i \underbrace{\left[-\mathcal{E}_{\theta_i} g_i\left(\theta_i, \alpha_i\right)\right] \alpha_i \widehat{\theta}_i}_{\text{direct effect}>0},$$

where $\mathcal{M}_i$ is the welfare multiplier for a sector-i specific shock:

$$\mathcal{M}_i \equiv \frac{\mathbf{e}_i^T \left(\mathbf{I}-\mathbf{A}\right)^{-1} \boldsymbol{\alpha}}{\mathbf{e}_i^T \boldsymbol{\alpha}}, \quad \text{with} \quad \mathbf{A} \equiv \left[\mathcal{E}_{\boldsymbol{\alpha}}\left(\mathbf{g}\right) \boldsymbol{\mathcal{E}}_{\mathbf{P}}\left(\mathbf{a}\right)\right]^T \in \mathbb{R}^{n\times n}. \tag{61}$$

Proof. See Appendix E. □

As in the two-sector case, the welfare multiplier (61) is a sufficient statistics that allows us to assess whether the intersectoral demand linkages amplify or attenuate the sectoral shock θ_i. Again, three cases may arise:

Case 1: When $\mathcal{M}_i > 1$, then the intersectoral demand linkages *amplify* the sector-specific shock.

Case 2: When $0 < \mathcal{M}_i < 1$, then the intersectoral demand linkages *attenuate* the sector-specific shock.

Case 3: Finally, when $\mathcal{M}_i < 0$ then the intersectoral linkages 'flip' the sign of the shock, thereby transforming "good shocks" into "bad outcomes" or, symmetrically, "bad shocks" into "good outcomes".

Note that our welfare multiplier (61) is closely related to amplification effects in (production) networks. Indeed, let $\lambda_i\left(\mathbf{A}\right)$, $i \in \{1, 2, \ldots, n\}$, be the eigenvalues of $\mathbf{A}$. If the elasticities of g_i with respect to α_i are not too large, so that $\rho\left(\mathbf{A}\right) \equiv \max_i \left|\lambda_i\left(\mathbf{A}\right)\right| < 1$, one can expand (61) as follows:

$$\mathcal{M}_i = 1 + \frac{\mathbf{e}_i^T \mathbf{A} \boldsymbol{\alpha}}{\mathbf{e}_i^T \boldsymbol{\alpha}} + \frac{\mathbf{e}_i^T \mathbf{A}^2 \boldsymbol{\alpha}}{\mathbf{e}_i^T \boldsymbol{\alpha}} + \ldots.$$

The foregoing expression highlights the similarity between our analysis and production network analysis: the welfare multiplier is the sum of the direct effect of the shock, the 'first-order' effect on the other sectors, the second-order effect of those sectors on the other sectors and so on. Our analysis is thus quite similar to that in, e.g., Baqaee and Fahri (2024, p.512), where the cost-share based Heterogenous-Agent Input-Output (HAIO) matrix $\widetilde{\mathbf{\Omega}}$ is nested in a Leontieff inverse $\left(\mathbf{I} - \widetilde{\mathbf{\Omega}}\right)^{-1}$. In our case, the expenditure-share based price-index elasticity matrix $\mathbf{A}$ is nested in a Leontieff inverse $(\mathbf{I} - \mathbf{A})^{-1}$.

Finally, one may wonder whether or not there are sufficient conditions that guarantee that $\mathcal{M}_i > 0$ holds. As formally shown in the two-sector case, if we assume Cobb-Douglas upper-tier preferences and CES lower-tier preferences with monopolistic competition, this is guaranteed under reasonable assumption on between- vs within sector elasticities of substitution. In the general case with arbitrary preferences and an arbitrary market structure, there seem to exist no clear-cut results. The only 'sufficient condition' for $\mathcal{M}_i > 0$ is that the elasticities $\mathcal{E}_\alpha$g are not too large, i.e., each sectoral-equilibrium price index is not too sensitive to the market size.

6 Conclusion

In this paper, we develop a multi-sector general equilibrium model that nests many of the approaches used in the applied literature. Using this model, we first derive a 'welfare multiplier' and establish precise conditions under which welfare-improving localized sector-specific shocks are magnified or dampened in the aggregate. The magnitude and the sign of the welfare multiplier crucially hinge on complementarity or substitutability between goods in consumers' preferences: when goods are gross complements, the welfare effects are always positive, though they may get dampened; whereas they may get amplified, dampened, or even reversed when goods are gross substitutes. The case of Cobb-Douglas preferences is a border-line scenario in which a welfare-improving shock in one sector is always welfare improving in general and of the same magnitude as the sectoral shock itself.

Second, we show that a particular combination of assumptions widely used in the literature—namely CES preferences and a monopolistically competitive market structure—give rise to specific results. In particular, these assumptions jointly guarantee that welfare gains always occur under reasonable assumptions on the elasticities of substitution of different tiers of preferences.

Third, we show that there exists a large class of homothetic non-CES preferences and market structures under which positive shocks are dampened as they propagate and where welfare losses may eventually occur. The reason is that monopolistically competitive sectors with homothetic non-CES preferences have prices that vary with sectoral market sizes which, under certain conditions, produces additional negative welfare effects. The same holds true when one considers oligopolistically competetitve sectors, even when preferences are CES.

Last, we show how our analysis relates to recent work by Baqaee and Fahri (2019, 2024) who investigate the transmission of localized microeconomic shocks to the macroeconomy in the presence of various distortions. Although our analysis is superficially similar to theirs, we zoom in on the role of intersectoral demand linkages on the consumers' side, whereas they study the effects of input-output linkages between industries. We also put more emphasis on the role of market structure in mediating the transmission of localized shocks. The next natural step for future work would be to combine both approaches and to investigate jointly the role of intersectoral effects stemming from transmission channels on both the demand and supply sides of the economy.

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Online appendix

A. Proof of Lemma 1

Proof. It suffices to prove that the demand functions

$$u_1(y, P_1, P_2) \equiv \frac{y}{P_1}\alpha\left(\frac{P_1}{y},\, \frac{P_2}{y}\right), \quad \text{and} \quad u_2(y, P_1, P_2) \equiv \frac{y}{P_2}\left[1-\alpha\left(\frac{P_1}{y},\, \frac{P_2}{y}\right)\right], \tag{A.1}$$

where $y > 0$ is income, satisfy the following properties: (i) $P_1 u_1(y, P_1, P_2) + P_2 u_2(y, P_1, P_2) = y$, i.e., the budget constraint holds; and (ii) the Slutsky matrix

$$\mathbf{S}(P_1, P_2, y) \equiv \begin{pmatrix} \frac{\partial u_1}{\partial P_1} + u_1 \frac{\partial u_1}{\partial y} & \frac{\partial u_1}{\partial P_2} + u_2 \frac{\partial u_1}{\partial y} \\ & \\ \frac{\partial u_2}{\partial P_1} + u_1 \frac{\partial u_2}{\partial y} & \frac{\partial u_2}{\partial P_2} + u_2 \frac{\partial 2}{\partial y} \end{pmatrix}$$

of the demand system (A.1) is symmetric and negative semidefinite.
By Antonelli's (1952) integrability theorem, equation (A.1) describes true Marshallian demands generated by some continuous, monotonic, and strictly quasi-concave utility if and only if (i) and (ii) hold. That the budget constraint is satisfied follows immediately from (A.1). Moreover, the demands (A.1) are homogeneous of degree zero in $(P_1, P_2,\, y)$. For the case of two goods, this is sufficient for $\mathbf{S}(P_1, P_2,\, y)$ to be symmetric (see, e.g., Jehle and Reny, 2011, Ch. 2). To prove that $\mathbf{S}(P_1, P_2,\, y)$ is negative semidefinite, observe that the price vector $\mathbf{p} \equiv (P_1, P_2)$ lies in the nullspace of the Slutsky matrix due to the budget constraint. Furthermore, the vector $\mathbf{e} \equiv (1,\, 0)$ always renders the quadratic form induced by $\mathbf{S}$ negative. Indeed, the $(1,1)$-entry of the Slutsky matrix is given by

$$s_{11} \equiv \frac{\partial u}{\partial P_1} + u \cdot \frac{\partial u}{\partial y} = -(1-\alpha)\left(\frac{\alpha y}{P_1^2} - \frac{1}{P_1}\frac{\partial \alpha}{\partial (P_1/y)}\right) - \frac{P_2}{P_1^2}\alpha\frac{\partial \alpha}{\partial (P_2/y)} < 0,$$

so we get

$$\mathbf{e}^T\mathbf{S}\mathbf{e} = s_{11} < 0. \tag{A.2}$$

Because the vectors $\mathbf{e} = (1,\, 0)$ and $\mathbf{P} = (P_1, P_2) \in \mathbb{R}^2_{++}$ form a basis in $\mathbb{R}^2$, there must exist coefficients θ_1 and θ_2 for any vector $\mathbf{h} = (h_1,\, h_2) \in \mathbb{R}^2$ such that $\mathbf{h} = \theta_1\mathbf{e} + \theta_2\mathbf{P}$. Computing $\mathbf{h}^T\mathbf{S}\mathbf{h}$, we get:

$$\mathbf{h}^T\mathbf{S}\mathbf{h} = \theta_1^2\mathbf{e}^T\mathbf{S}\mathbf{e} + 2\theta_1\theta_2\mathbf{e}^T\mathbf{S}\mathbf{P} + \theta_1^2\mathbf{P}^T\mathbf{S}\mathbf{P} = \theta_1^2\mathbf{e}^T\mathbf{S}\mathbf{e} = \theta_1^2 s_{11}.$$

Due to (A.2), we always have $\theta_1^2 s_{11} \leq 0$, whence $\mathbf{S}(P,\, P_2,\, y)$ is negative semidefinite. $\square$

B. Proof of Lemma 2

The elasticity of substitution $\gamma(P_1, P_2)$ between two goods is defined as follows:

$$\gamma(P_1, P_2) \equiv - \left.\frac{\mathrm{d}\ln(X_1/X_2)}{\mathrm{d}\ln(P_1/P_2)}\right|_{V(P_1,P_2)=\text{const}}, \tag{B.1}$$

where $V(P_1, P_2)$ is the indirect utility, and X_1 and X_2 are the Marshallian demands for goods 1 and 2, respectively:[25]

$$X_1(P_1, P_2) \equiv \frac{a(P_1, P_2)}{P_1}, \quad \text{and} \quad X_2(P_1, P_2) \equiv \frac{1 - a(P_1, P_2)}{P_2}. \tag{B.2}$$

Thus, (B.1) can be rewritten as

$$\gamma(P_1, P_2) \equiv 1 - \left.\frac{\mathrm{d}\ln[a/(1-a)]}{\mathrm{d}\ln(P_1/P_2)}\right|_{V(P_1,P_2)=\text{const}}. \tag{B.3}$$

Since $V(P_1, P_2)$ is held constant, it must be that $(\mathrm{d}P_1,\ \mathrm{d}P_2) \perp \nabla V$, i.e., price changes are orthogonal to the gradient ∇V. Furthermore, Roy's identity implies that $\nabla V \parallel (X_1, X_2)$, i.e., the gradient is parallel to the Marshallian demands. Combining these observations with (B.2) yields:

$$(\mathrm{d}P_1,\ \mathrm{d}P_2) \parallel \left(\frac{a(P_1, P_2)}{P_1},\ \frac{1 - a(P_1, P_2)}{P_2}\right),$$

which, in turn, implies

$$\begin{aligned}
\left.\frac{\mathrm{d}\ln[a/(1-a)]}{\mathrm{d}\ln(P_1/P_2)}\right|_{V(P_1,P_2)=\text{const}} &= \frac{P_1/P_2}{a/(1-a)} \left.\frac{\mathrm{d}\,(a/(1-a))}{\mathrm{d}(P_1/P_2)}\right|_{V(P_1,P_2)=\text{const}} \\
&= \frac{\dfrac{1}{(1-a)^2}\dfrac{\partial a}{\partial P_1}\mathrm{d}P_1 - \dfrac{1}{(1-a)^2}\dfrac{\partial(1-a)}{\partial P_2}\mathrm{d}P_2}{\dfrac{1}{P_2}\mathrm{d}P_1 - \dfrac{P_1}{P_2^2}\mathrm{d}P_2} \frac{P_1}{P_2}\frac{1 - a(P_1, P_2)}{a(P_1, P_2)} \\
&= \frac{\dfrac{1}{(1-a)^2}\dfrac{\partial a}{\partial P_1}\dfrac{1-a}{P_2} + \dfrac{1}{(1-a)^2}\dfrac{\partial(1-a)}{\partial P_2}\dfrac{a}{P_1}}{\dfrac{1-a}{P_2}\dfrac{1}{P_2} + \dfrac{P_1}{P_2^2}\dfrac{a}{P_1}} \frac{P_1}{P_2}\frac{1 - a(P_1, P_2)}{a(P_1, P_2)}.
\end{aligned}$$

After simplifications, we thus obtain

$$\frac{\mathrm{d}\ln[a/(1-a)]}{\mathrm{d}\ln(P_1/P_2)} = \mathcal{E}_{P_1}(a) + \mathcal{E}_{P_2}(1-a),$$

[25]Recall that income is the numéraire and that it does not change as any eventual profits go to absentee shareholders.

which we can plug into (B.3) to obtain (17). □

C. Proof of Lemma 3

Using (15), we have

$$\frac{\partial RHS(\alpha,\theta)}{\partial \alpha} = \frac{\partial a}{\partial P_1}\frac{\partial P_1}{\partial \alpha} + \frac{\partial (1-a)}{\partial P_2}\frac{\partial P_2}{\partial (1-\alpha)}.$$

Combining this expression with **(A2)** and (18)–(19) shows that $\partial RHS/\partial\alpha < 0$ under gross complements, while the opposite holds under gross substitutes. This proves part (i). To prove part (ii), we use (15) to compute

$$\frac{\partial RHS}{\partial \theta} = \frac{\partial a}{\partial P_1}\frac{\partial P_1}{\partial \theta}.$$

Combining this expression with **(A1)** and (18)–(19) shows that $\partial RHS/\partial\theta < 0$ under gross complements, while the opposite holds under gross substitutes. □

D. Welfare multiplier for a two-sector economy

We first apply the implicit function theorem to (13)–(14) to get:

$$\begin{pmatrix} 1-\frac{\partial P_1}{\partial \alpha}\frac{\partial a}{\partial P_1} & -\frac{\partial P_1}{\partial \alpha}\frac{\partial a}{\partial P_2} \\ -\frac{\partial P_2}{\partial (1-\alpha)}\frac{\partial (1-a)}{\partial P_1} & 1-\frac{\partial P_2}{\partial (1-\alpha)}\frac{\partial (1-a)}{\partial P_2} \end{pmatrix} \begin{pmatrix} \frac{\mathrm{d}P_1^*}{\mathrm{d}\theta} \\ \frac{\mathrm{d}P_2^*}{\mathrm{d}\theta} \end{pmatrix} = \begin{pmatrix} \frac{\partial P_1}{\partial \theta} \\ 0 \end{pmatrix}.$$

Solving this linear system for $(\mathrm{d}P^*/\mathrm{d}\theta,\ \mathrm{d}P_2^*/\mathrm{d}\theta)^T$ yields:

$$\begin{pmatrix} \frac{\mathrm{d}P^*}{\mathrm{d}\theta} \\ \frac{\mathrm{d}P_2^*}{\mathrm{d}\theta} \end{pmatrix} = \begin{pmatrix} 1-\frac{\partial P_1}{\partial \alpha}\frac{\partial a}{\partial P_1} & -\frac{\partial P_1}{\partial \alpha}\frac{\partial a}{\partial P_2} \\ -\frac{\partial P_2}{\partial (1-\alpha)}\frac{\partial (1-a)}{\partial P_1} & 1-\frac{\partial P_2}{\partial (1-\alpha)}\frac{\partial (1-a)}{\partial P_2} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial P_1}{\partial \theta} \\ 0 \end{pmatrix}.$$

By inverting the matrix, we obtain:

$$\begin{pmatrix} \frac{\mathrm{d}P_1^*}{\mathrm{d}\theta} \\ \frac{\mathrm{d}P_2^*}{\mathrm{d}\theta} \end{pmatrix} = \frac{\frac{\partial P_1}{\partial \theta}}{1-\left(\frac{\partial P_1}{\partial \alpha}\frac{\partial a}{\partial P_1} + \frac{\partial P_2}{\partial (1-\alpha)}\frac{\partial (1-a)}{\partial P_2}\right)} \begin{pmatrix} 1-\frac{\partial P_2}{\partial (1-\alpha)}\frac{\partial (1-a)}{\partial P_2} \\ \frac{\partial P_2}{\partial (1-\alpha)}\frac{\partial (1-a)}{\partial P_1} \end{pmatrix}. \tag{D.1}$$

Recall next that, by homotheticity of the lower-tier utilities, we have: $X_1 = \alpha/P_1$ and $X_2 = (1-\alpha)/P_2$. Using this relationship, and plugging the expressions (D.1) for $\mathrm{d}P_1^*/\mathrm{d}\theta$ and $\mathrm{d}P_2^*/\mathrm{d}\theta$

into (20) and, we get

$$\frac{\mathrm{d}E^*}{\mathrm{d}\theta} = \frac{\partial P_1}{\partial \theta} \frac{\frac{\alpha}{P_1}\left(1 - \frac{\partial P_2}{\partial(1-\alpha)}\frac{\partial(1-a)}{\partial P_2}\right) + \frac{1-\alpha}{P_2}\frac{\partial P_2}{\partial(1-\alpha)}\frac{\partial(1-a)}{\partial P}}{1 - \left(\frac{\partial P_1}{\partial \alpha}\frac{\partial a}{\partial P} + \frac{\partial P_2}{\partial(1-\alpha)}\frac{\partial(1-a)}{\partial P_2}\right)}. \tag{D.2}$$

Simplifying the numerator of the fraction in the right-hand side of (D.2) yields:

$$\begin{aligned} &\frac{\alpha}{P_1}\left(1 - \frac{\partial P_2}{\partial(1-\alpha)}\frac{\partial(1-a)}{\partial P_2}\right) + \frac{1-\alpha}{P_2}\frac{\partial P_2}{\partial(1-\alpha)}\frac{\partial(1-a)}{\partial P_1} \\ &\quad = \frac{\alpha}{P_1}\left(1 - \frac{1-\alpha}{P_2}\frac{\partial P_2}{\partial(1-\alpha)}\frac{\partial(1-a)}{\partial P_2}\frac{P_2}{1-\alpha} + \frac{1-\alpha}{P_2}\frac{\partial P_2}{\partial(1-\alpha)}\frac{\partial(1-a)}{\partial P_1}\frac{P_1}{1-\alpha}\frac{1-\alpha}{\alpha}\right) \\ &\quad = \frac{\alpha}{P_1}\left(1 - \mathcal{E}_{P_2}(1-a)\mathcal{E}_{1-\alpha}(P_2) - \mathcal{E}_{1-\alpha}(P_2)\mathcal{E}_{P_1}(a)\right). \end{aligned} \tag{D.3}$$

Note also that in equilibrium we have

$$\frac{\partial P_1}{\partial \alpha}\frac{\partial a}{\partial P_1} = \mathcal{E}_\alpha(P_1)\mathcal{E}_{P_1}(a), \quad \text{and} \quad \frac{\partial P_2}{\partial(1-\alpha)}\frac{\partial(1-a)}{\partial P_2} = \mathcal{E}_{P_2}(1-a)\mathcal{E}_{1-\alpha}(P_2), \tag{D.4}$$

so that the denominator in (D.2) becomes

$$1 - \frac{\partial P_1}{\partial \alpha}\frac{\partial a}{\partial P} - \frac{\partial P_2}{\partial(1-\alpha)}\frac{\partial(1-a)}{\partial P_2} = 1 - \mathcal{E}_\alpha(P_1)\mathcal{E}_P(a) - \mathcal{E}_{P_2}(1-a)\mathcal{E}_{1-\alpha}(P_2). \tag{D.5}$$

Plugging (D.3)–(D.5) into (D.2) completes the proof. □

E. Multisector welfare multiplier

From combining (56) with (59),

$$\widehat{\mathbf{P}} = [\mathcal{E}_{\boldsymbol{\theta}}(\mathbf{g})]\,\widehat{\boldsymbol{\theta}} + [\mathcal{E}_{\boldsymbol{\alpha}}(\mathbf{g})]\,[\boldsymbol{\mathcal{E}}_{\mathbf{P}}(\boldsymbol{a})]\,\widehat{\mathbf{P}}, \tag{F.1}$$

where $\widehat{\boldsymbol{\theta}}$ is the vector of percentage changes in $\boldsymbol{\theta}$. Solving (F.1) with respect to $\widehat{\mathbf{P}}$ yields:

$$\widehat{\mathbf{P}} = \left(\mathbf{I} - \mathbf{A}^T\right)^{-1}[\mathcal{E}_{\boldsymbol{\theta}}(\mathbf{g})]\,\widehat{\boldsymbol{\theta}}, \tag{F.2}$$

where $\mathbf{A}$ is the demand linkage matrix defined as follows:

$$\mathbf{A} := [\mathcal{E}_{\boldsymbol{\alpha}}(\mathbf{g})\,\boldsymbol{\mathcal{E}}_{\mathbf{P}}(\boldsymbol{a})]^T.$$

Plugging (F.2) into (60), we get:

$$\widehat{W} = -\boldsymbol{\alpha}^T \left(\mathbf{I} - \mathbf{A}^T\right)^{-1} \left[\mathcal{E}_{\boldsymbol{\theta}}(\mathbf{g})\right] \widehat{\boldsymbol{\theta}}. \tag{F.3}$$

Setting $\widehat{\boldsymbol{\theta}} = \widehat{\theta}_i \mathbf{e}_i$, we get:

$$\widehat{W} = -\left[\mathbf{e}_i^T (\mathbf{I} - \mathbf{A})^{-1} \boldsymbol{\alpha}\right] \left[-\mathcal{E}_{\theta_i} g_i\right] \widehat{\theta}_i = \mathcal{M}_i \underbrace{\alpha_i \left[-\mathcal{E}_{\theta_i} g_i\right] \widehat{\theta}_i}_{\text{direct effect}},$$

where $\mathcal{M}_i$ is given by (61). This completes the proof. $\square$